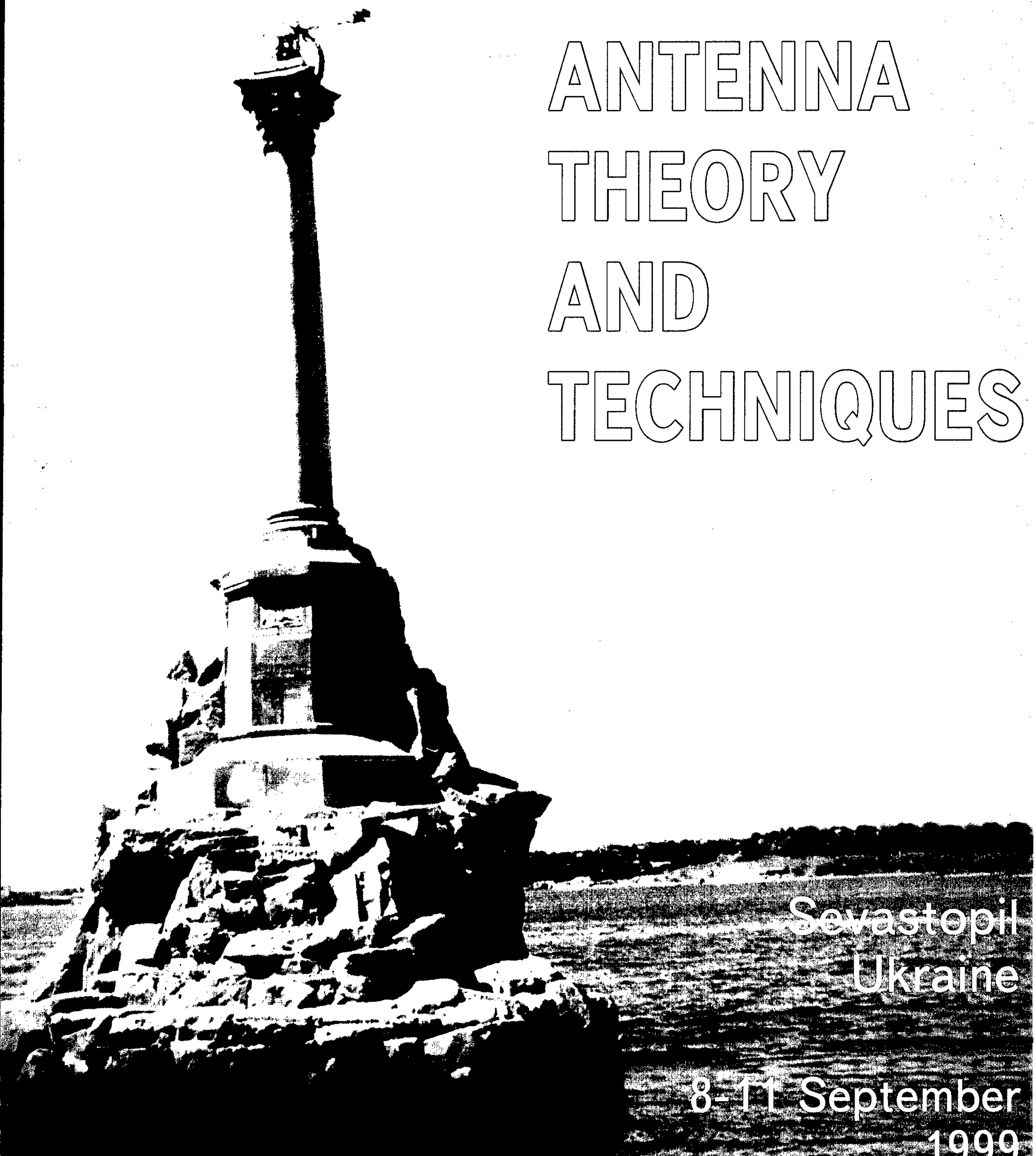
Proceedings of IIIrd International Conference

ANTENNA THEORY AND TECHNIQUES



Sevastopil
Ukraine

8-11 September
1999

Third International Conference on

Antenna Theory and Techniques

8-11 September 1999

Organizers

National Antenna Association of Ukraine

National Technical University of Ukraine "Kyiv Polytechnic Institute"

IEEE AP/MTT/AES/ED/LEO/GRS-SS East Ukraine Joint Chapter

Sevastopil State Technical University

Venue

Sevastopil State Technical University

Sevastopil, Ukraine

CO-ORGANIZERS AND CO-SPONSORS

STATE COMMITTEE OF UKRAINE ON SCIENCE AND INTELLECTUAL PROPERTY

KHARKIV TECHNICAL UNIVERSITY OF RADIO ELECTRONICS

TERA LTD. (KYIV)

COMPANY "VEBER" (SEVASTOPIL)

ROMSAT LTD. (KYIV)

TERNOPIL STATE DESIGN BUREAU "PROMIN" (TERNOPIL)

We wish to thank the following for their contribution to the success of this Conference:

IEEE REGION 8 OFFICE

US AIR FORCE EUROPEAN OFFICE OF AEROSPACE R&D

OFFICE OF NAVAL RESEARCH INTERNATIONAL FIELD OFFICE

Proceedings of the Third International Conference on Antenna Theory and Techniques
ISBN 966-622-012-1

This material is based upon work supported by the European Office of Aerospace Research and Development, Airforce Office of Scientific Research, Airforce Research Laboratory under Contract No. F 61775-99-WF071.

This work relates to Department of the Navy Grant N 00014-99-1-1033 issued by the Office of Naval Research International Field Office. The United States has a royalty-free license throughout the world in all copyrightable material contained herein.

ANTENNA ARRAYS

THE MATRIX MODELS OF DIGITAL ANTENNA ARRAYS WITH NONIDENTICAL CHANNELS

V. I. Slyusar

Central Research and Development Institute of Armament and Military Engineering
Kiev, Andruschenko Street, 4, e-mail: swadim@777.com.ua

The modern technology of radar and mobile communications systems is adaptive digital beam forming. When considering the multicoordinate digital beam forming in radar and communication systems with nonidentical channels of antenna arrays there is a problem of compact matrix record of the receiving channels responses. To solve the given problem it is proposed to operate with a special type of the matrices product, named by the author as "penetrated" and "generalized face-splitting" products.

According to the definition [1], for p×g-matrix A and p×gn-matrix B with p×g-blocks ($B=[B_n]$) their penetrated face-splitting product $A \boxdot B$ is the p×gn-block-matrix $[A \circ B_n]$, where "$\circ$" -a symbol of Adamar splitting, B_n — is a p×g-block of matrix B:

$$A \boxdot B=\left[A \circ B_1 \mid A \circ B_2 \mid \cdots \mid A \circ B_n \mid \cdots\right] \text{ or } A \boxdot B=\begin{bmatrix} A \circ B_1 \\ \hdashline A \circ B_2 \\ \hdashline \vdots \\ \hdashline A \circ B_n \\ \hdashline \vdots \end{bmatrix}.$$

The example:

$$A=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}, \quad B=\begin{bmatrix} B_1 \\ \hdashline B_2 \\ \hdashline B_3 \end{bmatrix}=$$

$$=\begin{bmatrix} b_{111} & b_{121} \\ b_{211} & b_{221} \\ b_{311} & b_{321} \\ \hdashline b_{112} & b_{122} \\ b_{212} & b_{222} \\ b_{312} & b_{322} \\ \hdashline b_{113} & b_{123} \\ b_{213} & b_{223} \\ b_{313} & b_{323} \end{bmatrix}, \quad A \boxdot B=\begin{bmatrix} a_{11} \cdot b_{111} & a_{12} \cdot b_{121} \\ a_{21} \cdot b_{211} & a_{22} \cdot b_{221} \\ a_{31} \cdot b_{311} & a_{32} \cdot b_{321} \\ \hdashline a_{11} \cdot b_{112} & a_{12} \cdot b_{122} \\ a_{21} \cdot b_{212} & a_{22} \cdot b_{222} \\ a_{31} \cdot b_{312} & a_{32} \cdot b_{322} \\ \hdashline a_{11} \cdot b_{113} & a_{12} \cdot b_{123} \\ a_{21} \cdot b_{213} & a_{22} \cdot b_{223} \\ a_{31} \cdot b_{313} & a_{32} \cdot b_{323} \end{bmatrix}.$$

As an example, the response of three-coordinate flat digital antenna array of R×R elements can be written down through penetrated face-splitting product of matrices as (without noise):

$$U=\dot{a} \cdot (Q \boxdot F)=\dot{a} \cdot \left[Q \circ F_1 \mid Q \circ F_2 \mid \cdots \mid Q \circ F_r \mid \cdots\right],$$

where $\dot{a}$ — is a complex signal amplitude,

$$Q=\begin{bmatrix} \dot{Q}_{11}(x,y) & \dot{Q}_{12}(x,y) & \cdots & \dot{Q}_{1R}(x,y) \\ \dot{Q}_{21}(x,y) & \dot{Q}_{22}(x,y) & \cdots & \dot{Q}_{2R}(x,y) \\ \vdots & \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y) & \dot{Q}_{R2}(x,y) & \cdots & \dot{Q}_{RR}(x,y) \end{bmatrix}$$

is the matrix of the directivity characteristics of primary channels in azimuth and elevation angle planes (can not be factorized),

$$F=\left[\begin{array}{ccc|c|ccc} \dot{F}_{111}(\omega) & \cdots & \dot{F}_{1R1}(\omega) & & \dot{F}_{11G}(\omega) & \cdots & \dot{F}_{1RG}(\omega) \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ \dot{F}_{R11}(\omega) & \cdots & \dot{F}_{RR1}(\omega) & & \dot{F}_{R1G}(\omega) & \cdots & \dot{F}_{RRG}(\omega) \end{array}\right]$$

is the block-matrix of amplitude-frequency characteristics meanings $\dot{F}_{nmg}(\omega)$ of G filters for R×R nonidentical receiving channels ($\dot{F}_{11g}(\omega) \neq \dot{F}_{rrg}(\omega)$);

$$Q \circ F_g=\begin{bmatrix} \dot{Q}_{11}(x,y)\dot{F}_{11g}(\omega) & \cdots & \dot{Q}_{1R}(x,y)\dot{F}_{1Rg}(\omega) \\ \dot{Q}_{21}(x,y)\dot{F}_{21g}(\omega) & \cdots & \dot{Q}_{2R}(x,y)\dot{F}_{2Rg}(\omega) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R1}(x,y)\dot{F}_{R1g}(\omega) & \cdots & \dot{Q}_{RR}(x,y)\dot{F}_{RRg}(\omega) \end{bmatrix},$$

U – block-matrix of voltages of the channels responses.

To select a single source on four coordinates (azimuth, elevation angle, frequency and range) the response of digital antenna array can be written down through generalized face-splitting product or generalized transposed face-splitting product (the theory of face-splitting products is presented in [1–4]). According to the definition, for block-matrices $A=[A_{ij}]$ and $B=[B_{ig}]$ with p×g- blocks their generalized face-splitting product $A \tilde{\square} B$ is the block-matrix

$$\left[A_{ij} \boxdot \left[B_{i1} \; B_{i2} \; \cdots \; B_{ig} \; \cdots\right]\right].$$

The example:

$$A \tilde{\square} B=\begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\square} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix}=$$

$$=\left[\begin{array}{c|c|c} A_{11} \boxdot [B_{11} \; \cdots \; B_{1G}] & \cdots & A_{1T} \boxdot [B_{11} \; \cdots \; B_{1G}] \\ A_{21} \boxdot [B_{21} \; \cdots \; B_{2G}] & \cdots & A_{2T} \boxdot [B_{21} \; \cdots \; B_{2G}] \\ \vdots & \vdots & \vdots \\ A_{P1} \boxdot [B_{P1} \; \cdots \; B_{PG}] & \cdots & A_{PT} \boxdot [B_{P1} \; \cdots \; B_{PG}] \end{array}\right].$$

The alternative to the above considered is a generalized transposed face-splitting block-matrices product:

$$A \tilde{\blacksquare} B = \left[A_{ij} \boxdot \begin{bmatrix} B_{1j} \\ B_{2j} \\ \vdots \\ B_{Gj} \end{bmatrix} \right].$$

For example, it can be written down:

$$A \tilde{\blacksquare} B = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1T} \\ A_{21} & A_{22} & \cdots & A_{2T} \\ \vdots & \vdots & \cdots & \vdots \\ A_{P1} & A_{P2} & \cdots & A_{PT} \end{bmatrix} \tilde{\blacksquare} \begin{bmatrix} B_{11} & B_{12} & \cdots & B_{1G} \\ B_{21} & B_{22} & \cdots & B_{2G} \\ \vdots & \vdots & \cdots & \vdots \\ B_{P1} & B_{P2} & \cdots & B_{PG} \end{bmatrix} =$$

$$= \left[\begin{array}{c:c:c:c} A_{11} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{12} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{1T} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \\ \hdashline \vdots & \vdots & \cdots & \vdots \\ \hdashline A_{P1} \boxdot \begin{bmatrix} B_{11} \\ B_{21} \\ \vdots \\ B_{P1} \end{bmatrix} & A_{P2} \boxdot \begin{bmatrix} B_{12} \\ B_{22} \\ \vdots \\ B_{P2} \end{bmatrix} & \cdots & A_{PT} \boxdot \begin{bmatrix} B_{1G} \\ B_{2G} \\ \vdots \\ B_{PG} \end{bmatrix} \end{array} \right].$$

The response of four-coordinate flat digital antenna array with R×R nonidentical channels can be present as (without noise):

$$U = \left(Q \boxdot \left\{S \,\tilde{\square}\, F\right\}\right) \cdot \dot{a} = Q \boxdot \left[S_1 \boxdot F \,\vdots\, S_2 \boxdot F \,\vdots \cdots \vdots\, S_T \boxdot F \right] \cdot \dot{a},$$

where

$$S = \left[\begin{array}{ccc:c:ccc} S_{111}(z) & \cdots & S_{1R1}(z) & & S_{11T}(z) & \cdots & S_{1RT}(z) \\ \vdots & \cdots & \vdots & \cdots & \vdots & \cdots & \vdots \\ S_{R11}(z) & \cdots & S_{RR1}(z) & & S_{R1T}(z) & \cdots & S_{RRT}(z) \end{array} \right]$$

is the matrix of the responses of single signal in T range gates (all channels have nonidentical radio impulse curve $S_{11t}(\omega) \neq S_{rrt}(\omega)$).

The alternate to considered above variant of analytical model of four-coordinate radar with flat digital antenna array is

$$U = \left(Q \boxdot \left\{\tilde{S} \,\tilde{\blacksquare}\, \tilde{F}\right\}\right) \cdot \dot{a} = Q \boxdot \left[\begin{array}{c} S_1 \boxdot \tilde{F} \\ \hdashline \vdots \\ \hdashline S_T \boxdot \tilde{F} \end{array} \right] \cdot \dot{a},$$

where

$$\tilde{S} = S^R = \left[S_1 \,\vdots \cdots \vdots\, S_T\right]^R = \left[\begin{array}{c} S_1 \\ \hdashline \vdots \\ \hdashline S_2 \end{array} \right] = \left[\begin{array}{ccc} S_{111}(z) & \cdots & S_{1R1}(z) \\ \vdots & \cdots & \vdots \\ S_{R11}(z) & \cdots & S_{RR1}(z) \\ \hdashline S_{11T}(z) & \cdots & S_{1RT}(z) \\ \vdots & \cdots & \vdots \\ S_{R1T}(z) & \cdots & S_{RRT}(z) \end{array} \right],$$

$$\tilde{F} = F^R = \left[F_1 \,\vdots \cdots \vdots\, F_G\right]^R = \left[\begin{array}{c} F_1 \\ \hdashline \vdots \\ \hdashline F_2 \end{array} \right] = \left[\begin{array}{ccc} F_{111}(\omega) & \cdots & F_{1R1}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R11}(\omega) & \cdots & F_{RR1}(\omega) \\ \hdashline F_{11G}(\omega) & \cdots & F_{1RG}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R1G}(\omega) & \cdots & F_{RRG}(\omega) \end{array} \right],$$

"R" is the symbol of block-rotation (this new block-matrix operation is proposed by author).

With the considered matrices models, on the basis of Neudecker's matrix derivative [3,4] an information Fischer's block-matrix, describing the accuracy of joint estimation of angular coordinates, range and frequency, is obtained:

$$I = \frac{1}{\sigma^2} \cdot \left[\begin{array}{c:c} P^T \cdot P & a^* \cdot P^T \cdot \dfrac{\partial P}{\partial Y} \\ \hdashline \dot{a} \cdot \left(\dfrac{\partial P}{\partial Y}\right)^T \cdot P & \left(\dfrac{\partial P}{\partial Y}\right)^T (\dot{a}a^* \cdot 1_{RRTG}) \dfrac{\partial P}{\partial Y} \end{array} \right],$$

where 1_{RRTG} — a unit matrix of dimension R×R×T×G, $\dfrac{\partial P}{\partial Y}$ — Neudecker's derivative of matrix P on vector Y formed of unknown estimations of angular coordinates, range and frequency of sources,

$$P = Q \boxdot \left\{S \,\tilde{\square}\, F \quad \text{or} \quad P = Q \boxdot \left\{\tilde{S} \,\tilde{\blacksquare}\, \tilde{F}\right.\right. .$$

To analyse multistatic radar systems the following matrix model (without noise) can be used:

$$U = \left(\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} \tilde{\square} \begin{bmatrix} S_{11} & \cdots & S_{T1} \\ \vdots & \vdots & \vdots \\ S_{1P} & \cdots & S_{TP} \end{bmatrix} \tilde{\square} \begin{bmatrix} F_{11} & \cdots & F_{G1} \\ \vdots & \vdots & \vdots \\ F_{1P} & \cdots & F_{GP} \end{bmatrix} \right) \cdot \dot{a},$$

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_P \end{bmatrix} = \left[\begin{array}{ccc} \dot{Q}_{111}(x,y) & \cdots & \dot{Q}_{1R1}(x,y) \\ \dot{Q}_{211}(x,y) & \cdots & \dot{Q}_{2R1}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R11}(x,y) & \cdots & \dot{Q}_{RR1}(x,y) \\ \hdashline \vdots & & \\ \hdashline \dot{Q}_{11P}(x,y) & \cdots & \dot{Q}_{1RP}(x,y) \\ \dot{Q}_{21P}(x,y) & \cdots & \dot{Q}_{2RP}(x,y) \\ \vdots & \vdots & \vdots \\ \dot{Q}_{R1P}(x,y) & \cdots & \dot{Q}_{RRP}(x,y) \end{array} \right],$$

$$S_{tp} = \begin{bmatrix} S_{11tp}(z) & \cdots & S_{1Rtp}(z) \\ \vdots & \cdots & \vdots \\ S_{R1tp}(z) & \cdots & S_{RRtp}(z) \end{bmatrix},$$

$$F_{gp} = \begin{bmatrix} F_{11gp}(\omega) & \cdots & F_{1Rgp}(\omega) \\ \vdots & \cdots & \vdots \\ F_{R1gp}(\omega) & \cdots & F_{RRgp}(\omega) \end{bmatrix},$$

$$U_{gtp} = \left(Q_p \circ S_{tp} \circ F_{gp}\right) \cdot \dot{a},$$

P is a number of radar position.

In the general case, for multiple signals the modeling concept, based on using of block generalized face-splitting product (symbol "$\tilde{\circledcirc}$") and block generalized transposed face-splitting product (symbol "$\tilde{\circledast}$") can be proposed. According to the definition,

$$A \tilde{\circledcirc} B = \left[A_{bg} \tilde{\square} B_{bk}\right]_{dn}, \quad A \tilde{\circledast} B = \left[A_{bg} \tilde{\blacksquare} B_{kg}\right]_{dn}.$$

The example:

$$A = \left[\begin{array}{cc:cc} A_{111} & A_{121} & A_{112} & A_{122} \\ A_{211} & A_{221} & A_{212} & A_{222} \end{array}\right],$$

$$B = \left[\begin{array}{cc:cc} B_{111} & B_{121} & B_{112} & B_{122} \\ B_{211} & B_{221} & B_{212} & B_{222} \end{array}\right], \quad A \tilde{\circledcirc} B =$$

$$= \left[\left[\begin{matrix} A_{111} & A_{121} \\ A_{211} & A_{221} \end{matrix}\right] \tilde{\square} \left[\begin{matrix} B_{111} & B_{121} \\ B_{211} & B_{221} \end{matrix}\right] \vdots \left[\begin{matrix} A_{112} & A_{122} \\ A_{212} & A_{222} \end{matrix}\right] \tilde{\square} \left[\begin{matrix} B_{112} & B_{122} \\ B_{212} & B_{222} \end{matrix}\right]\right],$$

$$A \tilde{\circledast} B =$$

$$= \left[\left[\begin{matrix} A_{111} & A_{121} \\ A_{211} & A_{221} \end{matrix}\right] \tilde{\blacksquare} \left[\begin{matrix} B_{111} & B_{121} \\ B_{211} & B_{221} \end{matrix}\right] \vdots \left[\begin{matrix} A_{112} & A_{122} \\ A_{212} & A_{222} \end{matrix}\right] \tilde{\blacksquare} \left[\begin{matrix} B_{112} & B_{122} \\ B_{212} & B_{222} \end{matrix}\right]\right].$$

The model of four-coordinate radar with flat digital antenna array in multisignal case can be written as:

$$U = \left(Q \tilde{\circledast} \left(S \tilde{\circledast} F\right)\right)\left(A \otimes 1_R\right),$$

where A is the vector of complex amplitudes of signals of Ì sources,

$$Q = \left[Q_1 \ Q_2 \cdots Q_M\right], \quad Q_m = \left[\begin{matrix} Q_{11}(x_m, y_m) & \cdots & Q_{1R}(x_m, y_m) \\ \vdots & \cdots & \vdots \\ Q_{R1}(x_m, y_m) & \cdots & Q_{RR}(x_m, y_m) \end{matrix}\right]$$

$$S = \left[S_1 \ S_2 \ \cdots \ S_M\right],$$

$$S_m = \left[\begin{matrix} S_{111}(z_m) & \cdots & S_{1R1}(z_m) \\ \vdots & \cdots & \vdots \\ S_{R11}(z_m) & \cdots & S_{RR1}(z_m) \\ \hdashline \vdots & & \\ S_{11T}(z_m) & \cdots & S_{1RT}(z_m) \\ \vdots & \cdots & \vdots \\ S_{R1T}(z_m) & \cdots & S_{RRT}(z_m) \end{matrix}\right],$$

$$F = \left[F_1 \ \ F_2 \cdots F_M\right], F_m = \left[\begin{matrix} F_{111}(\omega_m) & \cdots & F_{1R1}(\omega_m) \\ \vdots & \cdots & \vdots \\ F_{R11}(\omega_m) & \cdots & F_{RR1}(\omega_m) \\ \hdashline \vdots & & \\ F_{11G}(\omega_m) & \cdots & F_{1RG}(\omega_m) \\ \vdots & \cdots & \vdots \\ F_{R1G}(\omega_m) & \cdots & F_{RRG}(\omega_m) \end{matrix}\right],$$

1_R is a unit matrix of dimension R; $\otimes$ — symbol of Kronecker's- products of matrixes.

REFERENCES

1. Slyusar V. I. The family of the face-splitting matrices products and its characteristics// Kibernetika i sistemny analiz. - 1999.- ¹ 3. - to be published [in Russian].

2. Slyusar V. I. New operations of matrices product for applications of radars, in Proc. Direct and Inverse Problems of Electromagnetic and Acoustic Wave Theory (DIPED-97), Lviv, September 15-17, 1997, P. 73-74 [in Russian].

3. Slyusar V. I. Accuracy of linear digital antenna array at joint estimation of range and angular coordinate of M sources, in Proc. ICATT—97, Kyiv, May 1997.- P. 110 – 111.

4. Slyusar V. I. The face-splitting matrix products in radar applications.// Radioelectronika. -1998. - ¹ 3. - P. 71 - 75 [in Russian].