60 years to the theory of electrically small antennas.
Some results

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Transceiver modules for mobile communications, receivers of satellite navigation systems and other microelectronics applications get smaller and smaller. That is why downsizing of antenna devices and their integration in one package with systems-on-a-chip is a vital problem. A reasonable question of a limit for antenna downsizing arises. It was attempted to be solved in the 40th of the previous century, when a need for small board antennas for jet aircrafts and rocket engineering emerged. In 2007 we celebrate 60 years since the first theoretical publication which contained reasoning for fundamental limits on parameters of so-called electrically small antennas (ESA) [1]. On the eve of this event it is time to summarize the results of the 60-years way passed by theorists and antenna designers in the understanding of maximum limitations on realization of effective antenna solutions.

On November 11 1946 Harold A. Wheeler published an article [1] which was the first one to connect the determination of electrically small receiving antenna with its maximal size. He suggested denoting such antennas as those with the size less than half wavelength of the received electromagnetic oscillations or $\frac{l}{2p}$. In the other words, ESA are antennas which satisfy the following condition:

$$ka < 1, \quad (1)$$

where $k = \frac{2p}{l}$ - represents the wave number, l denotes wavelength, a stands for the radius of reference sphere, enclosing the maximal size of a dipole antenna or the radius of the corresponding hemisphere for the case of monopole (Fig.1). Without any losses we may interpret expression (1) as unstrict inequality, i.e. assuming that $ka \le 1$.

For the reference sphere circumscribed around ESA, Wheeler introduced a notion of “radian sphere” widely accepted in theory. The surface of this sphere is often interpreted as a reference boundary between near and far field, created by the transmitting ESA.

Wheeler was the first to explore the electrical properties of elementary ESA (electric and magnetic dipoles) as well as their fundamental limitations [2]. To characterize these limitations he introduced a notion of “radiation power factor”, which in its physical essence is inverse to antenna’s quality factor and equivalent to its impedance bandwidth (i.e. bandwidth measured at the VSWR level usually set to 2). Values of the radiation power factor for electrical (p_e) and magnetic (p_m) dipoles were obtained by Wheeler using their equivalent electrical circuits (Fig.2):

$$p_e = \frac{G_e}{wC}, \; p_m = \frac{R_m}{wL},$$

where G_e stands for equivalent conductance, shunting the capacitance of electrical dipole.

In this case in general the values of p_e and p_m are limited by a “cylindrical” volume of ESA, equal to the volume Vol of cylinder circumscribed around it in correspondence with the following expression [1]:

$$p_{e(m)} > \frac{k^3 Vol}{6p} = \frac{k^3 Ab}{6p} = \frac{4p^2}{3l^3} Vol, \quad (2)$$

where A represents the cylinder’s base area, and b stands for its height, which is equal to the dipole’s one.

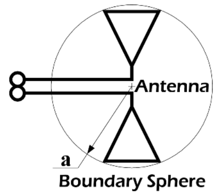
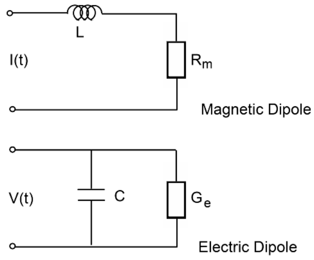


Fig.1 To the definition of ESA



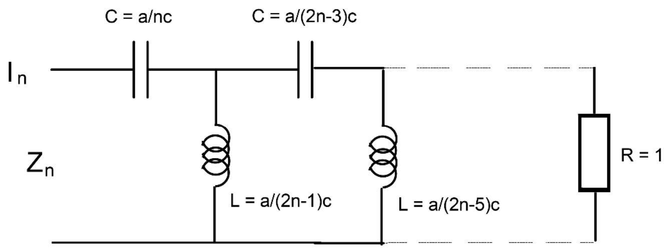
Equivalent electrical circuits of electrical (a) and magnetic (b) dipoles

Fig.2

In 1948 Chu introduces the fundamental quality factor limits for transmitting ESA with linear and circular polarizations, inscribed into the radian sphere [3]. At the same time he regarded ESA as the source of spherical electromagnetic waves, located in free space. Amplitude and phase distributions of current along antenna were assumed to be uniform. Under such assumptions the quality factor was characterized by the ration of electromagnetic energy accumulated inside the radian sphere surrounding antenna to the power radiated beyond it P [3]. In essence, during analysis Chu used only the prevailing component of electromagnetic field (either electrical or magnetic), which allowed defining the quality factor in the following form [2]:

$$Q = \begin{cases} \dfrac{2wW_e}{P}, & W_e > W_m, \\ \dfrac{2wW_m}{P}, & W_m > W_e. \end{cases}, \tag{3}$$

where W_e or W_m represents the energy accumulated either by electrical or magnetic field (depending on the dipole type), w stands for radial frequency of electromagnetic oscillations.



Equivalent RLC circuit of ESA in the case TMn wave used by Chu: a denotes the radian sphere's radius, c stands for light velocity.

Fig.3

Approximating the radiator of spherical waves by equivalent RLC circuits (each RLC circuit corresponds to its own radiation mode) (Fig.3), Chu obtained approximate expressions for the lower boundary of the ESA quality factor in case of vertical linear and circular polarizations. As a result he demonstrated that, when $ka > n$ (n is the sequence number of # mode propagating in antenna), the quality factor of antenna with circular polarization is approximately two time less than in case of antennas with linear polarization [3].

However the results obtained for transmitting antennas may be extended for receiving antennas only with some degree of probability. Assumption of sphericity of the transmitted waves is not completely valid for (the front of electromagnetic waves received in the far-field region is almost flat) and does not allow strictly using the reciprocity theorem with respect to Chu's theoretical premises.

Despite the complexity of calculations, Chu managed to obtain the upper boundary for the gain of a directed ESA [3]:

$$G \approx \frac{2}{p} \cdot \frac{2pa}{l} = \frac{4a}{l}, \tag{4}$$

as well as to build graphs for the boundary value of quality factor Q_n. However for their qualitative characteristic it worth limiting ourselves to the known limits of the second order Henkel functions, which are a part of Chu's boundary relations and when $ka \to 0$ have the following form [4]:

$$H_n(ka) \approx \frac{2i}{p} \frac{1}{(ka)^n}, \tag{5}$$

where i stands for imaginary unit.

Analyzing asymptotic behavior of the Henkel functions (5) we may conclude that ESA possesses minimal quality factor when receiving the lowest mode. With decreasing the antenna's dimensions its boundary quality factor grows rapidly.

Use of equivalent RLC circuits for antenna analysis allows making a transition from the minimal quality factor to the maximal achievable antenna's impedance bandwidth [2]:

$$\partial f = \frac{VSWR - 1}{Q \cdot \sqrt{VSWR}}.$$

Since in practice usually VSWR=2 is used when determining the impedance bandwidth ∂f of antennas, we can easily obtain [2]

$$\partial f_{VSWR=2} = \frac{1}{Q \cdot \sqrt{2}}.$$

Thus, the smaller the size of ESA, the greater its quality factor and the more difficult it is to provide wideband reception of signals. The connection of the quality factor with antenna's bandwidth became the basis for further development of ESA theory in the direction of their boundary quality factor research. However, the radicalism if this step and complexity of the used mathematical apparatus became a barrier on the way of fast popularization of the new approach and its practical application by other researchers. Only in 1960 Harrington specified Chu's theory for ESA with rotating polarization by describing their quality factor with the following approximating dependence [5, 6]:

$$Q_c = \frac{1}{2}\left[\frac{1 + 3k^2a^2}{k^3a^3\left(1 + k^2a^2\right)}\right]. \tag{6}$$

This is one of the first analytical expressions for the quality factor boundary. Harrington also noted the existence of the upper boundary gain of ESA which he obtained in the following form [7]:

$$G = k^2a^2 + 2ka.$$

Considering Chu's conclusion regarding double difference between the quality factor value of antennas with circular and linear polarizations for the lowest radiation mode and $ka > 1$ and using the results of work [5] we can easily obtain an approximation of Chu's boundary for ESA with vertical polarization:

$$Q = \frac{1 + 3k^2a^2}{k^3a^3\left(1 + k^2a^2\right)}. \tag{7}$$

This expression appears in many publications as the Chu-Harrington boundary [2, 8].

It is worth noting another consequence of expressions (6) and (7). When $ka << 1$ the value of k^2a^2 may be neglected with respect to unity and then we obtain:
for circular polarization

$$Q_c \approx \frac{1}{2k^3a^3}, \tag{8}$$

for vertical polarization

$$Q \approx \frac{1}{k^3a^3}. \tag{9}$$

Expressing (9) in terms of volume of the radian sphere, circumscribed around dipoles, considered by Wheeler in [1], in correspondence with expression (2) we may obtain alternative boundary $Q_{\min}^{-1} = \frac{6p^2}{l^3}Vol = \frac{3k^3}{4p}Vol$.

In this case the minimum of the quality factor $Q_{\min}$ is 4.5 times less than the Wheeler's boundary, which is reasonable and may be explained by increased volume of the radian sphere of the cylindrical ESA volume.

Due to simplicity, expressions (8) and (9) are often used during practical design of ESA as the Chu boundary approximation. When $ka = 0{,}1$, their error is insignificant with respect to expressions (6) and (7) and to the other theoretical formulas used for calculating the boundary values of Q. These relations essentially close the first stage of making ESA theory, when approximating of antennas by means of their equivalent RLC circuits was used to research the boundary parameters.

The start of the **second stage** in the making of ESA theory was laid by Robert E. Collin and S. Rothschild with their method of calculating minimal quality factor Q without using the equivalent electrical circuits [9]. Considering the present day view it was a serious step in the development of ESA theory, which is as important as works [1] and [3]. To specify the interpretation of the quality factor the authors suggested using the total energy accumulated near ESA, equal to the sum of its electrical and magnetic components, in the numerator of expression (3):

$$Q = \frac{w(W_e + W_m)}{P}. \tag{10}$$

After performing integration of the complex Pointing vector the authors [9] obtained new analytical relations for the quality factors of ESA with vertical polarization using its first three radiation modes:

$$Q_1 = \frac{1}{k^3a^3} + \frac{1}{ka}, \tag{11}$$

$$Q_2 = \frac{6}{k^3a^3} + \frac{3}{ka} + \frac{18}{k^5a^5}, \tag{12}$$

$$Q_3 = \frac{21}{k^3a^3} + \frac{6}{ka} + \frac{135}{k^5a^5} + \frac{675}{k^7a^7}. \tag{13}$$

Besides, the methodology of analytical calculation of the total reactive component's energy and the power of radiated electromagnetic field for the first time allowed obtaining a general expression for the boundary quality factor in the case of cylindrical, but not only spherical electromagnetic waves. Along with this, relation (11) did not come from any of the previously obtained results of ESA theory and contradicted with the quality factor boundary, defined by expression (7). Thus, a question arose: whose evaluation of the boundary quality factor is closer to reality? The search of answer to this question represents the essence of the second stage of developing ESA theory. Research took the way of analytical description of the field's parameters.

The direction, laid by (9), continues to develop today. However it took five years before Fante broadened the results of this work and generalized them for multimode antennas in 1969 [10].

R.C . Hansen's article [11] may be considered as the symbol of the second stage of making ESA theory. In this work the author, concluding the results of a general antenna theory, still operates with boundary quality factor, defined by expression (7) [11]:

$$Q = \frac{1+3k^2a^2}{k^3a^3\left(1+k^2a^2\right)}.$$

In the contrast to the existing at that moment serious works on the application of the electromagnetic field theory for analysis of ESA he decides to use the methodology of Chu's equivalent circuits again. This step marked a split among researchers into the group of supporters of Chu-Harrington's results and those to support the idea of their revision.

With the deployment of mobile telephony in the 90^{th} of the previous century interest to ESA substantially increased, which gave impetus to reviewing the existing by that time theoretical conceptions. One of the reasons for that was the discovered by McLean mistake in mathematical calculations of the fundamental boundary (7) for the quality factor of ESA [6]. As a result of recheck he obtained a more accurate expression for the quality factor boundary:

$$Q = \frac{1+2k^2a^2}{k^3a^3\left(1+k^2a^2\right)}, \tag{14}$$

differing from the previous one by coefficient 2 in nominator, but not 3.

Such a definite response to the doubts in correctness of results, expressed in their article by R. Collin and S. Rothschild, obtained by Chu supporters introduced the third stage of making ESA theory. The nominator of the expression (14) corrected by McLean may be represented in terms of sum:

$$Q = \frac{1+2k^2a^2}{k^3a^3\left(1+k^2a^2\right)} = \frac{1+k^2a^2+k^2a^2}{k^3a^3\left(1+k^2a^2\right)} = \frac{1}{k^3a^3} + \frac{1}{ka\left(1+k^2a^2\right)}.$$

It is easy to notice that when $ka \ll 1$ the value of k^2a^2 being of the second infinitesmall order if compared to unity in the denominator of the second summand may be neglected. As a result we obtain:

$$Q \approx \frac{1}{k^3a^3} + \frac{1}{ka}, \tag{15}$$

which is identically equal to formula (11). McLean suggested using this relation, which proved the correctness of Collin and Rothschild results, to determine the corrected fundamental boundary of the quality factor of a freely located in space ESA.

From the curves graphed for the quality factor values (7), (9), (14) and (15) it follows that the boundaries determined from expressions (7) and (15) coincide at the point $ka = 1$. When $ka < 1$, the Chu-Harrington boundary (7) passes a little above the dependency (15) (Fig.4). Thus, the mathematical error in the Chu-Harrington dependency (7) in the quality factor calculations of ESA does not exceed 12.5% with respect to McLean's approximation (15). Assumption that $ka \ll 1$, allowing to use an approximate formula (15), is valid for $ka < 0{,}6$.

For circular polarization McLean also obtained a more accurate, if compared to formula (6) by Harrington, dependency of the quality factor:

$$Q_c = \frac{1}{2}\left[\frac{2}{ka} + \frac{1}{k^3a^3}\right] = \frac{1}{ka} + \frac{1}{2k^3a^3}. \tag{16}$$

However the necessity of even more accurate evaluation of the boundary capabilities of ESA stimulated further research with the use of computation technologies. Progress in numerical modeling of antennas allowed theorists to apply a new powerful apparatus for investigating ESA properties. By means of the time-domain description of electrodynamics in antennas E.D. Caswell with co-authors used a more accurate interpretation of the antennas quality factor [12] through the peak value of accumulated reactive energy of electromagnetic field [2]:

$$Q=\frac{w\cdot\max[W_e+W_m]}{P}. \qquad (17)$$

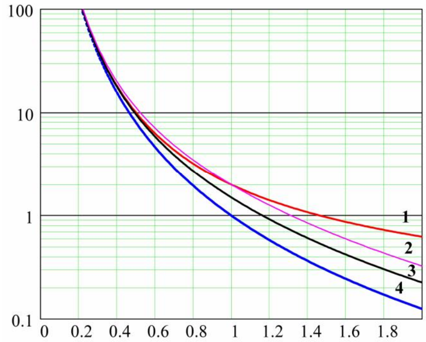


Fig.4 A family of quality factor boundaries: curves 1, 2, 3 and 4 correspond to formulas (15), (7), (14) and (9)

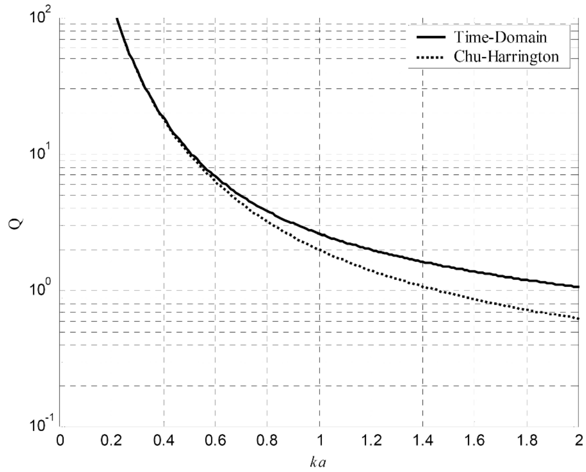


Fig.5 Quality factor boundaries defined by formulas (19) and (15) under 100% efficiency of antenna

Computation of the peak energy for the single-mode radiation of arbitrary polarization allowed reformulating the fundamental boundary, previously set by Wheeler [12, 13]:

$$Q=\frac{1}{2k^3a^3}+\frac{1}{ka}+\sqrt{\frac{1}{4k^6a^6}+\frac{1}{k^2a^2}}. \qquad (18)$$

For the sake of fairness we should note that D.M. Grimes and C.A. Grimes have the priority in calculating expression (18), who published an identically equal result in May of 1998 in a little different form [14]:

$$Q=\frac{1}{2k^3a^3}\left(1+\sqrt{1+4k^4a^4}\right)+\frac{1}{ka}. \qquad (19)$$

It should be noted that when $ka \ll 1$ in (19) we may neglect the value of k^4a^4 of the fourth infinitesmall order with respect to unity and obtain the same result, as in the simplified formula (15). This once more proves the validity of McLean’s expression (15).

Form the graph of dependencies (15) and (19), corresponding to the theoretical case of 100% efficiency of antenna, it follows that the boundary defined by expression (19) passes above McLeans’s boundary. At the point $ka=1$ the quality factor values differ by 1.31 times. Grimes points out that in contrast to the boundary, obtained by Chu during calculation of the resonant case, boundary (19) is applicable in the absence of resonance as well, when there is no strict equality $W_m=W_e$.

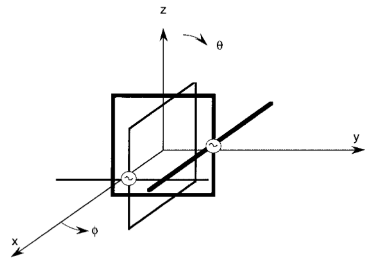


Fig. 6 Combined double-dipole ESA [15, 16]

In the following works Grimes, who made a significant contribution to the making of the theory of fundamental boundary of ESA quality factor, studied the boundary quality factor of a system, composed of two elementary dipoles (Fig.6) as a function of the signal's phases in dipoles [15, 16]. He demonstrated that under co-phase feeding of the dipoles pair the validity of expression (14) and the other similar relation for ESA is retained. In the case of receiving waves with circular polarization their total quality factor may be up to 20 times less than the boundary quality factor of Chu-McLean for the radian sphere's radius, corresponding to ka = 0,23. Thus work of Grimes in cooperation with the other specialists opens possibilities of searching for constructions of multi-element antennas with a smaller quality factor boundary, than in the case of single ESA, which are consequently more wideband. However, not all specialist share Grimes's optimism regarding the possibility of reaching almost zero quality factor of electrically small antennas systems [17].

The necessity of taking antenna's efficiency (E) into account in the relation for fundamental quality factor boundary should be noted. The thing is that the mentioned above expressions (4)-(19) correspond to 100% efficiency of antenna, since during their calculations an ideal antenna without losses was considered. It is clear that such assumption is never valid in practice. To account for the possible Joule losses in ESA the relation [18, 19] may be used

$$Q_r = Q \cdot h \,, \qquad (20)$$

where Q represents the lower quality factor boundary for the ideal conditions, Q_r stands for the quality factor boundary in the presence of Joule losses, and h denotes efficiency of ESA.

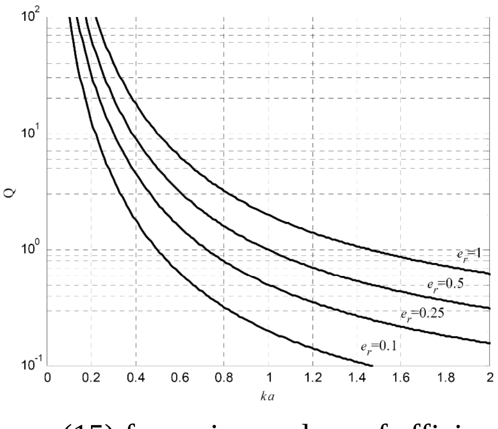


Fig.7 Calculated dependency (15) for various values of efficiency considering weighting according to formula (20)

Analysis of expression (20) reveals that variation of efficiency from 100% leads to a decrease of the antenna's quality factor and, as a consequence, to subsidence of the fundamental boundary graphs (Fig. 7). Thus, widening the bandwidth of ESA is accomplished by decreasing antenna's efficiency. A similar result may be obtained in the case of polarization losses as well, while in this case antenna's polarization efficiency h_p should be used in expression (20) as the weighting multiplier:

$$Q_r = Q \cdot h_p .$$

Accounting for Joule and polarization losses leads to more complicated dependencies. However in general such approach appears to be rather simplified, since the dependency of efficiency on ka is ignored. The effect of efficiency instability with decreasing the size of ESA becomes more vivid when there are some screens or the other even dielectric objects near antenna.

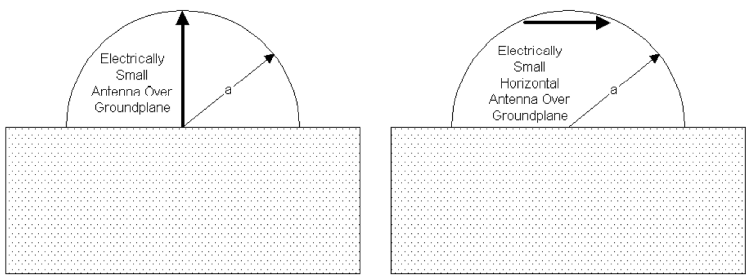
Research of the boundary quality factor value for ESA [20] placed over a conducting screen (Fig.8) [7] performed in 2001 represents significant interest. It was stated that the screen has almost no influence of the quality factor of the vertical antenna radiating waved with vertical polarization. That is why McLean's formula (14) or its alternatives for the free space is valid for such antenna. The case of horizontal radiation of ESA (parallel to the screen's surface) is completely different. Forced application of horizontally polarized waved in this case leads to redistribution of electromagnetic field energy in the antenna's neighborhood. This process is accompanied by the growth of accumulated reactive energy between the antenna and the screen. As a result the quality factor of this antenna significantly increases, while its bandwidth narrows by six and more times (depending on the radial sphere's size) if compared to the vertically placed antenna [7].

The majority of values of the quality factor boundary are obtained as a result of theoretical calculations. In cases of slight deviations from the calculations' assumptions they are not valid. However designers require not ephemeral evaluations from their point of view, but specific practically realizable boundaries, determined by a set of parameters of real antennas. Due to this fact the peculiarity of the present fourth stage in making ESA theory lays in the search of practically realizable boundaries with maximal accounting for all physical effects in antenna calculations. The beginning of this stage was marked by the work published in 2003, in which a calculation of the quality factor's boundary value with accounting for sine current distribution law in ESA's conductor is performed [21]. Previously when similar relations were attempted to be obtained a hypothesis of the uniform distribution of induced currents in antenna was accepted. According to [21], the quality factor of a small-sized antenna may be described by the following relation:

$$Q = \frac{\int_{pL/l}^{\infty} |E_n(u)|^2 \, du}{\int_{0}^{pL/l} |E_n(u)|^2 \, du}, \tag{21}$$

where

$$E_n(u) = \frac{\cos u - \cos\left(\frac{bL}{2}\right)}{\left[1 - \cos\left(\frac{bL}{2}\right)\right]\sqrt{1 - \left(\frac{2u}{bL}\right)^2}} . \qquad (22)\ [21]$$



Vertical (a) and horizontal (b) placement of vibrator ESA over conducting screen

Fig.8

Numerical integration of expression (21) with consideration of (22) allowed graphing a dependency for Q (Fig.9).

Comparison of dependency (21) with the experimental data shows that this boundary is not strict as well and may be broken when the current distribution law approaches uniform behavior. This fact is illustrated on the example of the Bow-tie antenna (Fig.10a) and the end-loaded dipole (Fig.10b) [21]. Current on the ends of the Bow-tie antenna has zero value. This explains good agreement of the corresponding Q values with those predicted by a theory described in [21]. To the contrary the current distribution of the end-loaded dipole is more uniform, that is why the Q values in this case are located closed to the boundary defined by expression (15). The greater the size of the end-loaded dipole, the more obvious this effect is.

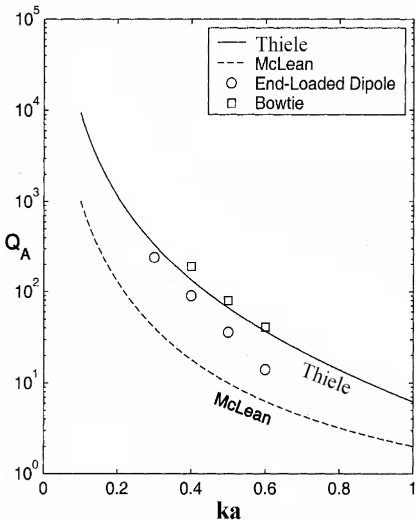


Fig.9 Relation of the Thiele boundary to the fundamental boundary of formulas (11) and (15) [21]

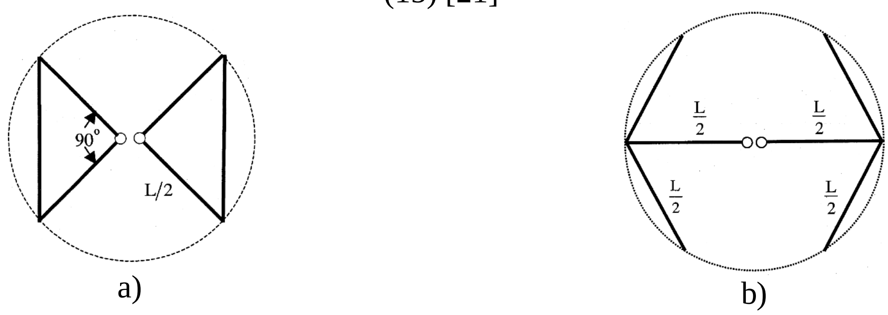


Fig.10 Bow-tie antenna (a) and end-loaded dipole (b) inscribed in circumference with diameter L

Thus, considering the results of work [21] we may conclude that in order to increase the accuracy of calculating the boundary quality factor values all nuances of the current distribution in the specific antenna type should be accounted for. In fact there is a quality factor boundary for each current distribution law, while bandwidth is maximal in case of uniform current profile of ESA.

In the case of nonsymmetrical ESA a sphere encloses greater volume than ellipsoid. The "wasted" volume significantly distorts the value of the theoretical Q boundary. This fact made the researchers refuse from the concept of the Wheeler's radian sphere in favor of a stretched spheroid (ellipsoid). Such ellipsoid in the case of vivid differences in transverse-longitudinal dimensions of ESA should enclose the space surrounding antenna [22]. Besides, the results of numerous experiments showed that the quality factor of a long thin dipole is greater than that for the wide one, while the bandwidth of the fixed-height dipole ESA grows with increasing transversal size of the dipole [22].

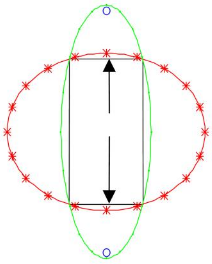


Fig.11 Types of bounding surfaces in the case of straight dipole

We should note that in ESA theory non-spherical volumes enclosing a dipole were rarely considered [9, 22], i.e. the idea of using a stretched spheroid is rather innovative for ESA theory. R.C. Adams and P.M. Hansen managed to obtain accurate analytical dependencies for calculating the quality factor of a dipole antenna under various parameters of ellipsoidal boundary [22]. To do that they used the traditional to ESA theory quality factor definition, i.e. the relation of energy *W*, accumulated within the limited volume of ellipsoidal surface, to power *P(u)*, radiated through this surface to the far-field region of the antenna (radial coordinate $u \to \infty$). The corresponding expression for the quality factor has the following form [22]:

$$Q_e = \frac{3}{4b^3}\left[\frac{u_m}{u_m^2-1} - \frac{1}{2u_m} + 0{,}25\left(u_m^{-2}-1\right)\ln\left(\frac{u_m+1}{u_m-1}\right)\right], \qquad (23)$$

where $b = \frac{p \cdot d}{l} = \frac{k \cdot d}{2} = \frac{w \cdot d}{2c}$ denotes the electrical size parameter of ESA in the spherical coordinates system (w stands for radial frequency, c represents light velocity, *d* means distance between ellipsoid's focuses). Small values of *β* are equivalent to a small electrical size of the dipole.

However the results obtained by the authors [22] do not have such general character as the results of a series of the previous authors. Due to non-orthogonal behavior of electromagnetic field's modes analytical relations of the minimal quality factor *Q* are valid only when the lowest radiation mode is contained in the radiated field.

A dipole antenna with length *L* and radius *r*, considered by Adams and Hansen, may be inscribed in a sphere, a cylinder or an ellipsoid (Fig.11). The cylindrical volume of such antenna is $\pi L r^2$, while the minimal volume of the sphere, circumscribed around the cylinder of this size, is equal [23]

$$Vol_{сф} = (4p/3)\left((L/2)^2 + r^2\right)^{3/2} \approx pL^3/6 + pr^2L\,. \qquad (24)$$

Approximate value in the right part of the equality corresponds to a thin cylinder (*L/2* >> *r*). As follows, an extra volume, enclosed by the sphere, i.e. the volume beyond the boundaries of the cylinder amounts to $pL^3/6$. The volume of ellipsoid circumscribed around the cylinder, according to [23], has the following form:

$$Vol_{эл} = (4p/3)r^2 b_e \big/ \left(1 - (0{,}5L/b_e)^2\right). \qquad (25)$$

The length of the largest semiaxis of the ellipse b_e, to which the minimal ellipsoid's volume corresponds,

$$Vol_{эл\,\min} = pr^2 L\sqrt{3}\,, \qquad (26)$$

is equal $L\sqrt{3}/2$.

From the comparison of the mentioned volumes of bounding surfaces (24)-(26) it follows, that if transversal size *r* of a dipole is significantly less than *L/2*, then the energy near ESA, accumulated within ellipsoid is much less, than the one inside the radian sphere. The parameters of ellipse considered in [22] correspond to the minimal volume of ellipsoid, enclosing the cylinder, which represent a dipole.

Under equal height of antennas the quality factor value Q for the stretched spheroid is greater than for the sphere. This perfectly agrees with the experimental results, which show that the selection of a thinner structure of antenna leads to larger quality factor Q and better selectivity of the system.

It is important to note that a transition from spherical approximation of the near-field region to spheroidal should not be accompanied by the requirement of equal volumes of these shapes, which would cause corresponding change of ESA height. For practically important cases the fixed value of dipole's height, its invariance to the law, which approximates the topology of the bounding surface, is of great interest. For such limitation on the change of ESA parameters the following boundary evaluation for electrical dipole, inscribed in ellipsoid, was obtained [22]:

$$Q_e = 0{,}75\left(\frac{l}{2pb_e}\right)^3 u_m^3\left[\frac{u_m}{u_m^2-1}-\frac{1}{2u_m}+0{,}25\left(u_m^{-2}-1\right)\ln\left(\frac{u_m+1}{u_m-1}\right)\right]. \quad (27)$$

To compare (27) with the results obtained for the spherical model of the bounding surface, enclosing antenna, it was suggested to equate the largest semiaxis of ellipsoid to the radius of radian sphere, i.e. $b_e = a$ [22]. As a result the value of the boundary quality factor, evaluated using formula (27), for instance, with respect to the boundary (9) possesses the following form:

$$\frac{Q_e}{Q_{sphera}} = 0{,}75u_m^3\left[\frac{u_m}{u_m^2-1}-\frac{1}{2u_m}+0{,}25\left(u_m^{-2}-1\right)\ln\left(\frac{u_m+1}{u_m-1}\right)\right]. \quad (28)$$

With increasing axial relation exceeding of the quality factor boundary, defined by (27), with respect to the boundary (9) increases (Fig.12). As u_m grows, and consequently the axial relation decreases, the value on the square parenthesis of formula (28) approaches $4/(3u_m^3)$, while the whole ratio approaches unity.

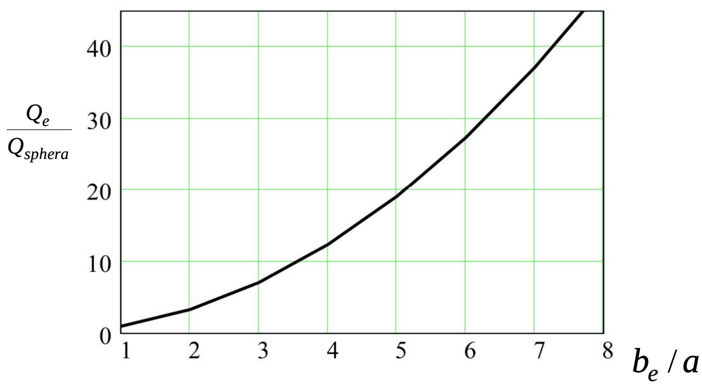


Fig.12

The disadvantage of work by Adams and Hansen [22] lies in the absence of specific instructions on the selection of the ratio between larger and smaller semiaxis of the bounding ellipsoid. The suggested way of connecting the parameters of ellipsoid to the size of a cylinder, circumscribed around ESA, is inapplicable in the case when the length of the larger semiaxis of ellipsoid coincides with antenna height. As an option, in this case we can connect the calculation of the spheroid's axial relation for the specific construction of ESA with the result of experimental verification of its quality factor. However such comparisons are absent in [22]. In the case of fractal antennas in wire and microstrip versions with vivid transversal-longitudinal asymmetry we recommend using the value of fractional size, which corresponds to the geometrical shape, as the value of axial relation of the bounding ellipsoid.

In general the results of [22] lead to a thought on the necessity to adapt the shape of bounding surface of ESA to the specific construction. In this case two approaches are possible. The first one expects adjusting the bounding surface to the geometrical shape of antenna, copying its fragments at the fixed distance from antenna. This approach is extremely complex for analytical calculation except for some simplest configurations of ESA similar to those considered by Wheeler [1]. However the problem of evaluating the electrical and magnetic components of electromagnetic field's energy using numerical calculation methods may be solved using computer modeling.

An alternative way expects accounting for the directional properties of ESA, specifically its radiation pattern (RP) obtained analytically and experimentally, during the selection of the bounding surface. In this case, for instance, the quality factor evaluation of dipole ESA by using the three-dimensional radiation pattern of antenna as a boundary (Fig.13). In this case non-uniform character of energy distribution of the electrical field component of dipole with its minimization near transversal vibrator's axis is accounted for. During a transition to higher resonances the shape of the boundary should change in correspondence with the changing configuration of the normalized RP. Analytical solution of such problem requires generalizing the apparatus of spherical wave functions for the corresponding topological boundary shapes. Possibly for a number of simpler RP, for instance, cardio-sphere, such solutions may be obtained in analytical form in the closest future. The main thing is to formulate the transformations of the space metric, corresponding to the stretched spheroid or the radian sphere, into the topology of the three-dimensional cardioid and similar boundary shapes. For more complex topological shapes numerical solutions of the problem should be used. It is important for these approaches to lead to comparable results. This would allow verifying the validity of the obtained ESA quality factor evaluations before transitioning to experimental research of the antenna constructions.

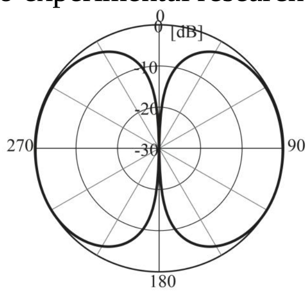


Fig.13 Cross-section of the three-dimensional radiation pattern of a straight dipole

The work [22] opens wide perspectives for further improvement of calculating the parameters of electromagnetic field of ESA with various geometries of bounding surfaces. This would allow obtaining calculation relations, applicable for practical synthesis of ESA with the most complex configurations.

We can't help mentioning serious difficulties, connected with the use of modern ESA theory to calculate quality factors of wideband antennas. It was not an accident that Johnson J.H. Wang pointed out groundlessness of the Chu theory to such applications and concluded the possibility of using all the previous ESA theory only to resonant antennas [24, 25]. Wang made this conclusion on the basis of analyzing the properties of a patented spiral printed antennas with the bandwidth 1-10 GHz. Though dimensions of the Wang's antennas do not satisfy the ESA definition even at the lower boundary of pass band (ka=1,597), nevertheless the obtained results allowed Wang to question the validity of the boundary formulas for region ka>1. The main difficulty lies in the interpretation of the quality factor notion. So, during the comparison of experimental results for the spiral printed antenna Wang substituted the frequency of the lower boundary of the pass band to formula (14) instead of the central pass band frequency [24]. This is explained by the fact that calculations of the ka value based on the central pass band frequency, as expected by the Chu-McLean boundary, give absurd values with negative overlapping coefficient. The possible reason of such lack of correspondence of the spiral printer antenna parameters and the theory of the fundamental quality factor boundary lies in the operation of its radial segments similarly to the Uda-Yagi antenna. Thus, in essence the considered antenna may be approximated by multi-vibrator antenna array of transversal radiation in each radial direction, which was mentioned in Wang's works. In this case, as Grimes showed, the validity of the fundamental boundaries, applicable to a single vibrator, is broken and a reasonable widening of the multi-element antenna bandwidth occurs [14, 15]. However, in spite of reasonability of such hypothesis, it requires verification. The Wang paradox needs to be solved before further development of ESA theory. The theory of “zero” quality factor introduced by Grimes may be used for this purpose.

The considered peculiarities of ESA gave birth to a series of serious problems of their design, caused by the fact that with decreasing size of antenna system its efficiency rapidly decreases and difficulties of matching ESA in non-resonant modes with signal sources (receivers) arise. We shall tell how researchers coped with these problems and on the most interesting ways of design efficient ESA in the following issues of the journal.

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