CONTENTS

VOLUME 41 NUMBER 5 1998

RADIOELECTRONICS AND COMMUNICATIONS SYSTEMS

Brief Communications

Radioelectronics and Communications Systems
Vol. 41, No. 5, pp. 30–33, 1998

Izvestiya VUZ. Radioelektronika
Vol. 41, No. 5, pp. 43–47, 1998
UDC 621.396.967

VARIATION OF THE ANALOG-TO-DIGITAL CONVERTER DIGITIZATION USING THE SIGNAL OF KNOWN FREQUENCY

V. I. Slyusar

Kiev, Ukraine

The paper proposes the methods of changing the ADC digitization period using a harmonic signal; their accuracy properties have been analyzed.

When using ADC boards for entering analog data into a PC there is often a necessity to have accurate measurements of the digitization period, especially when the ADC clock is shaped by the PC central processor and entirely depends on the speed of the bus interface of the specific mother board. To shape a strictly periodic ADC start the processor interruption is turned off but then it is impossible to measure the digitization interval without having precision measuring instruments. A similar problem may emerge also in the case of the ADC boards containing its own clock. A manufacturer often rounds up the conversion time value to the nearest integer.

To overcome these difficulties a method to measure the ADC clock period obtained from [1] is recommended. As applied to the problem being solved it is necessary to apply a harmonic signal of the known frequency to the ADC input and digitize it. Signal voltage samples stored in the program buffer then are subject to processing by the "three-sample" window sliding on the array according to the expression:

$$\Delta t = \frac{1}{2\pi f} \arccos \frac{\sum\limits_{n=1}^{N} U_{2n}(U_{1n} + U_{3n})}{2\sum\limits_{n=1}^{N} U_{2n}^2} \qquad (1)$$

where Δt is the sought-for digitization interval, f is the known test-signal frequency, U_{1n}, U_{2n}, U_{3n} are the voltages of the first, second, and third samples being processed in the nth position of the sliding window, N is the number of triads being averaged (the total number of the movements of the processing window).

The sliding pitch with the buildup of sample triads can be random, however, (if AFC of the analog channel has a sufficiently wide band to ignore noise correlation) here it is convenient to assign as equal to one ADC clock. The same is also true with respect to the interval between the triad samples. When thinning the signal sampling in (1) one should properly adjust the value f. Being optimal in terms of minimizing the mean-square error of measurement, procedure (1) ensures the accuracy of estimating the interval Δt proportional to the signal-to-noise ratio and the frequency of the test-signal under condition that the value of the latter does not exceed the quarter of the digitization frequency.

To achieve the potential accuracy of the measurements we may refuse the averaging of the triads of the samples having preferred the processing of the whole R-component sampling of voltages using the method of maximum likelihood. The gist of the synthesis of such a measuring procedure of ignoring the initial phase of the test-signal boils down the minimization of the functional:

$$F = \sum_{s=1}^{R} \{U_s - a\cos[\omega\,\Delta t\,(s-1)]\}^2 \tag{2}$$

where a is the signal amplitude, $\omega = 2\pi f$, U_s is the voltage of the ADC sth sampling.

Since minimum (2) occurs under the same conditions of the maximum of the value $F_m = a\sum_{s=1}^{R} U_s\cos[\omega\,\Delta t\,(s-1)]$, the sought-for procedure of determining Δt can be obtained having substituted the estimate of the test-effect amplitude satisfying the equation $\partial F/\partial a = 0$.

As a result, the value Δt will be determined by the exhaustive search of its possible estimates in the course of the maximization if the function:

$$\tilde{F}_m = \left\{\sum_{s=1}^{R} U_s\cos[\omega\,\Delta t\,(s-1)]\right\}^2 \left\{\sum_{s=1}^{R} \cos^2[\omega\,\Delta t\,(s-1)]\right\}^{-1}. \tag{3}$$

The limiting achievable accuracy of estimating can be characterized by the lower Cramer-Rao bound. The second particular derivatives of functional (2) were obtained for the derivation of the Cramer-Rao and the meanvalues for these particular derivatives were determined. As a result of the conversion of the respective Fisher information matrix the variance of the estimate of the digitization period $\sigma^2_{\Delta t}$ will be expressed in the form:

$$\sigma^2_{\Delta t} \geq \frac{\sigma_n^2}{a^2\omega^2}\left[\sum_{s=1}^{R}(s-1)^2\sin^2[\omega\,\Delta t\,(s-1)] - \frac{\left\{\sum_{s=1}^{R}(s-1)\sin[2\,\omega\,\Delta t\,(s-1)]\right\}^2}{4\sum_{s=1}^{R}\cos^2[\omega\,\Delta t\,(s-1)]}\right]^{-1} \tag{4}$$

where σ_n^2 is noise variance in one sample.

Analysis of relationship (4) indicates that the highest accuracy of measurement occurs with $\omega\,\Delta t = \frac{\pi}{2}(2n-1)$ $(n = 1, 2, \ldots)$, i.e. when the digitization frequency is four-fold as the frequency of the test-signal. In this case:

$$\sigma^2_{\Delta t} \geq \frac{6\,\sigma_n^2}{a^2\omega^2 R\,[R^2-1]}. \tag{5}$$

Formulas (4) and (5) do not take into account correlation of noises, the signal initial phase, ADC nonlinearity, noises of quantization and digitization, and instability of clock frequency.

To eliminate the bias of the estimate of the digitization period caused by the absence of information on the initial phase we have, instead of (2), to minimize the functional:

$$Q = \sum_{s=1}^{R} \{U_s - a^c\cos p_s + a^s\sin p_s\}^2 = \min \tag{6}$$

where $p_s = \omega\,\Delta t\,(s-1)$, $a^c = a\cos\varphi$, $a^s = a\sin\varphi$, φ is the initial phase.

Having solved the system of equations

$$\frac{\partial Q}{\partial a^c} = 0;\quad \frac{\partial Q}{\partial a^s} = 0$$

we will obtain the estimates of the quadratic components of the signal amplitude:

$$\hat{a}^c = \left[\sum_{s=1}^{R} \sin^2 p_s D_1 - \frac{1}{2} \sum_{s=1}^{R} \sin 2 p_s D_2 \right] D^{-1},$$

$$\hat{a}^s = \left[-\sum_{s=1}^{R} \cos^2 p_s D_2 + \frac{1}{2} \sum_{s=1}^{R} \sin 2 p_s D_1 \right] D^{-1}, \qquad (7)$$

where

$$D_1 = \sum_{s=1}^{R} U_s \cos p_s, \; D_2 = \sum_{s=1}^{R} U_s \sin p_s,$$

$$D = \left[\sum_{s=1}^{R} \sin^2 p_s \right] \left[\sum_{s=1}^{R} \cos^2 p_s \right] - \frac{1}{4} \left[\sum_{s=1}^{R} \sin 2 p_s \right]^2.$$

Analyzing condition (6) as in the case of functional (2) it is not difficult to make a conclusion on the possibility of its replacement by the equivalent criterion:

$$Q_m = a^c \sum_{s=1}^{R} U_s \cos p_s - a^s \sum_{s=1}^{R} U_s \sin p_s = \max.$$

The subsequent substitution into this relationship of estimates (7) of the series of transformations leads to the compact result:

$$Q_m = \frac{\sum_{s=1}^{R} \{\sin p_s D_1 - \cos p_s D_2\}^2}{D} = \max. \qquad (8)$$

In terms of feasibility it will be meaningful to transform the relationship obtained to the form similar to that shown in [3]:

$$Q_m = \frac{D_1^2 f_1 + D_2^2 f_2 - D_1 D_2 f_3}{D} = \max, \qquad (9)$$

where

$$f_1 = \sum_{s=1}^{R} \cos^2 p_s, f_2 = \sum_{s=1}^{R} \sin^2 p_s, f_3 = \sum_{s=1}^{R} \sin 2 p_s.$$

It is essential that in contrast to [3] frequency deviation is absent in expression (9), the denomination is taken into account and exhaustive search is performed using the unknown Δt.

As the results of the experiments depend on the ADC word length, the values of averaging and stability of digitization frequency according to (9) measurement accuracy Δt of the order of several nanoseconds may be obtained. For instance, on a 12-bit ADC using the two-volt harmonic signal of frequency 10 kHz the digitization period of 3.407 μs was measured at error ±2 ns. To this end the sampling of 4,000 signal samplings were used. A G3-110 low-frequency precision oscillator with the discretization of frequency setting 0.01 Hz.

Among other factors affecting the accuracy of method (9) as before it is necessary to indicate the signal-to-noise ratio and the frequency of the test-effect. In addition, extra dependence of the variance Δt on the initial phase of the harmonic signal φ appears. As could be expected the lower Cramer-Rao bound for (9) turns out to be larger than (4) and with $\omega \Delta t = \pi/2$, $\varphi = 0$ we will obtain:

$$\sigma^2_{\Delta t} \geq \frac{\sigma^2_n}{a^2 \omega^2} \left[\sum_{s=1}^{R} (s-1)^2 \sin^2\left[\frac{\pi}{2}(s-1)\right] - \frac{\left(\sum_{s=1}^{R} (s-1) \sin^2\left[\frac{\pi}{2}(s-1)\right]\right)^2}{\sum_{s=1}^{R} \sin^2\left[\frac{\pi}{2}(s-1)\right]} \right]^{-1},$$

or with the account of [2] for even R:

$$\sigma^2_{\Delta t} \geq \frac{6\,\sigma^2_n}{a^2 \omega^2 R\,[0.25 R^2 - 1]}$$

which is clearly larger than (5).

In conclusion we will note that the initial definition Δt by (1) and (3) with the subsequent perfection of the result within the framework of procedures (8) or (9) by assigning the frequency of the test-signal according to condition $\omega \, \Delta t = \pi(2n-1)/2$ may be considered as a version of the measuring procedure. To rule out the influence of the initial phase of the signal on the accuracy of the unbiased estimate Δt it is expedient to go to the complex representation of the voltage samples, for example, based on the Hilbert discrete transform.

REFERENCES

1. V. I. Slyusar, V. I. Pokrovskii, V. F. Sakhno, et al., "Method of measuring the frequency of harmonic oscillations". The RF Patent application for invention no. 5062480/09 (042802) of Sept. 18, 1992. IPC[6] G01R23/00. Positive decision of VNIIGPE on the issue of the patent of 27 September 1995.

2. I. S. Gradshtein and I. M. Ryzhik, Tables of Integrals, Sums, Series, and Products [in Russian], Nauka, Moscow, 1971.

3. V. I. Slyusar, V. I. Pokrovskii, and V. F. Sakhno, "Digital method of optimal reception of linear-frequency-modulated pulses", Patent no. 2042956, the Russian Federation, IPC G01 S7/285, 13/10. Publ. In BI no. 24, 1995, p. 228.

25 February 1997