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ACCURACY IN MEASURING ANGULAR COORDINATES BY A LINEAR DIGITAL ANTENNA ARRAY WITH NONIDENTICAL RECEIVING CHANNELS

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We determined the Cramer-Rao bounds for the variances of errors in estimating angular coordinates of point sources by a linear digital antenna array having known characteristics of receiving channels' directivity. The article also considers the influence on the measurement accuracy of directivity characteristics' identity and their dependence on the source bearing.

The paper offers a great number of methods for measuring the parameters of radiation sources including their angular coordinates. Under these conditions, as is noted in [1], the expressions for limit variance measurement errors acquired great significance since they make it possible to arrange the advantages of most competing measuring procedures.

The objective of the article is determination of the Cramner–Rao bounds characterizing the angular accuracy of the digital antenna array (DAA) with nonidentical directivity characteristics (DC) of the receiving channels. Let us consider a linear equidistant DAA of K elements one of which is chosen as a reference one. Assuming the directivity characteristics of all the receiving channels as complex we will write down the DAA response to a point sources of a flat electromagnetic wave in the form of a set of K voltages:

$$\dot{U}_k = U_k^c + j \cdot U_k^s = \dot{a} \cdot \dot{F}_k \cdot \exp(j \cdot x_k) + \dot{n}_k =$$
$$= [a^c \cdot F_k^c - a^s \cdot F_k^s] \cos x_k - \sin x_k \; [a^s \cdot F_k^c + a^c \cdot F_k^s] + j \times$$
$$\times \left\{ [a^c \cdot F_k^c - a^s \cdot F_k^s] \cdot \sin x_k + [a^s \cdot F_k^c + a^c \cdot F_k^s] \cos x_k \right\} + \dot{n}_k, \qquad (1)$$

where $\dot{F}_k = F_k^c + j \cdot F_k^s$ is the complex DC of the kth receiving channel in the direction x normalized with respect to the DC module of the reference channel; $\dot{a} = a^c + j \cdot a^s$ is the signal complex amplitude in the reference channel; $x_k = x \cdot (k - z)$; z is the point coordinate on the antenna aperture chosen as a phase center of the array; $x = \frac{2\pi}{\lambda} \cdot d \cdot \sin\theta$ is the generalized angular coordinate of the source; λ is the radiation wavelength; d is the array step; θ is the angle between the direction to the source and the normal to the array; $\dot{n}_k$ is the complex noise value in the kth channel.

Having assigned the Gaussian noise model we will assume that they are uncorrelated, have zero averages and identical variances of quadrature components in a channel taken individually.

To estimate the potential accuracy when measuring the generalized coordinate x we will use the logarithmic equivalent of the likelihood functional which at the accuracy to the constant multiplier C may be put down in the form:

$$L = \ln C - \frac{1}{2} \cdot \sigma_n^{-2} \cdot \sum_{k=1}^{K} \frac{1}{p_k^2} \cdot [\, \{ U_k^c - [a^c \cdot F_k^c - a^s \cdot F_k^s] \cdot \cos x_k +$$
$$+ [a^s \cdot F_k^c + a^c \cdot F_k^s] \cdot \sin x_k \}^2 + \{ U_k^s - [a^c \cdot F_k^c - a^s \cdot F_k^s] \cdot \sin x_k - [a^s \cdot F_k^c + a^c \cdot F_k^s] \cdot \cos x_k \}^2 \,],$$

where p_k^2 are the coefficients taking into account the spread of variances of noises in the DAA receiving channels with respect to the variance of noise σ_n^2 in the reference channel.

In the case the characteristic $\dot{F}_k$ depends on the directivity to the radiation source, the formation of the Fisher information matrix implies differentiating $\dot{F}_k$ with respect to the coordinate x which is natural under the condition that such is possible. Calculations lead to the following expression for the variance:

$$\sigma_x^2 \geq \frac{\sigma_n^2}{a^2} \cdot \left\{ \sum_{k=1}^{K} \frac{(F_k^{c'} - (k-z) \cdot F_k^s)^2 + (F_k^{s'} + (k-z) \cdot F_k^c)^2}{p_k^2} - Q \cdot \left[\sum_{k=1}^{K} \frac{F_k^{c^2} + F_k^{s^2}}{p_k^2} \right]^{-1} \right\}^{-1}, \tag{2}$$

where

$$Q = \left(\sum_{k=1}^{K} \frac{F_k^c \cdot F_k^{c'} + F_k^s \cdot F_k^{s'}}{p_k^2} \right)^2 + \left(\sum_{k=1}^{K} \frac{F_k^c \cdot F_k^{s'} - F_k^s \cdot F_k^{c'} + (k-z)\,(F_k^{c^2} + F_k^{s^2})}{p_k^2} \right)^2.$$

Relationship (2) in terms of the most general positions characterizes the accuracy of direction finding procedure boiling down to the search for the maximum of the function:

$$L_M = \left[\widetilde{U}^{c^2} + \widetilde{U}^{s^2} \right] \cdot \left(\sum_{k=1}^{K} \left[F_k^{c^2} + F_k^{s^2} \right] \cdot p_k^{-2} \right)^{-1} = \max, \tag{3}$$

where

$$\widetilde{U}^c = \sum_{k=1}^{K} p_k^{-2} \times \left\{ U_k^c \cdot \left[F_k^c \cdot \cos x_k - F_k^s \cdot \sin x_k \right] + U_k^s \cdot \left[F_k^s \cdot \cos x_k + F_k^c \cdot \sin x_k \right] \right\};$$

$$\widetilde{U}^s = \sum_{k=1}^{K} p_k^{-2} \times \left\{ U_k^s \cdot \left[F_k^c \cdot \cos x_k - F_k^s \cdot \sin x_k \right] - U_k^c \cdot \left[F_k^s \cdot \cos x_k + F_k^c \cdot \sin x_k \right] \right\}.$$

As for the algorithm oriented at the material of directivity characteristics, then for it the following is true:

$$\sigma_x^2 \geq \frac{\sigma_n^2}{a^2} \cdot \{ D + B \}^{-1}, \tag{4}$$

where

$$D = \sum_{k=1}^{K} \frac{(k-z)^2 F_k^2}{p_k^2} - \frac{\left(\sum_{k=1}^{K} \frac{(k-z) F_k^2}{p_k^2} \right)^2}{\sum_{k=1}^{K} F_k^2 \cdot p_k^{-2}};$$

$$B = \sum_{k=1}^{K} \frac{\left(F_k' \right)^2}{p_k^2} - \frac{\left(\sum_{k=1}^{K} \frac{F_k \cdot F_k'}{p_k^2} \right)^2}{\sum_{k=1}^{K} F_k^2 \cdot p_k^{-2}}.$$

Naturally, expression (4) may be obtained from (2), it is convenient for analysis. In particular, according to the Bunyakovskii–Cauchy inequality we may state that for any directivity characteristics F_k differentiated, the value $B \geq 0$.

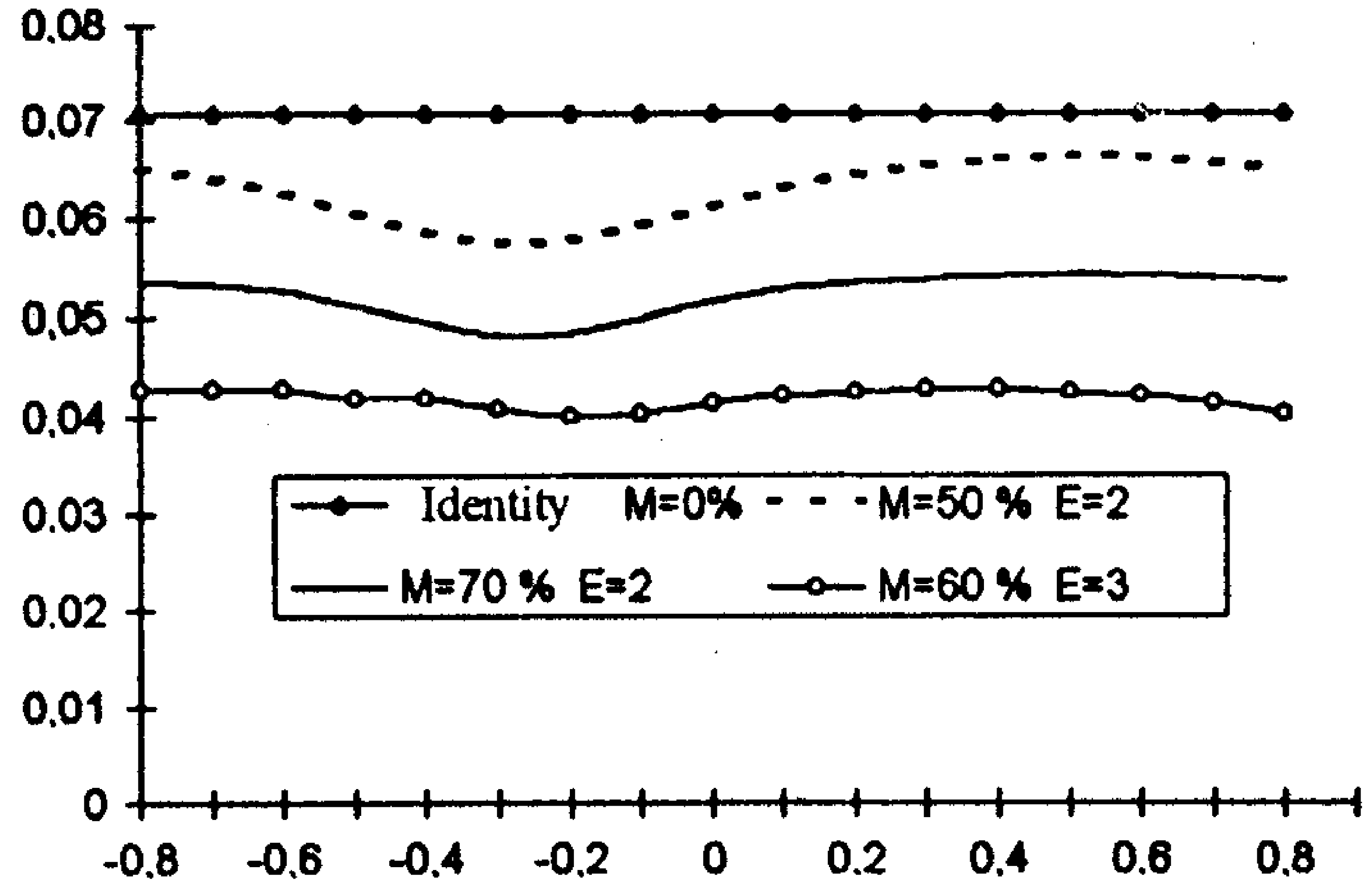

Fig. 1

Therefore, in the case $\frac{F_k}{F'_k} \neq \frac{F_m}{F'_m}$ with any other equal conditions, the direction finding procedure of type (3) oriented at arbitrary partial characteristics of the receiving channels in DAA may provide an advantage in accuracy compared with the algorithms operating invariant ones with respect to the directivity at the DC source.

From the analysis of the value B it follows that the said advantage in accuracy will be greater, the smaller the value of the sum $\sum_{k=1}^{K} F'_k F_k$, given the fixed value of the other part of denominator (4). This is achieved by the increase in nonidentity of partial DCs in the sector of interest in such a way that $\sum_{k=1}^{K} F'_k F_k \to 0$. Naturally, that in this case F_k are assumed as accurately known.

The result obtained may be confirmed by another interpretation. In accordance with expression (4), procedure (3) makes it possible to take the bearing of subnoise signals whose intensity the lowest, the greater is nonidentity of the characteristics in the assigned direction and their state of indenture determined by the derivatives F'_k. Here this is given a rigorous theoretical explanation.

Thus, to increase the range of a radar based on DAA there is an alternative for increasing the transmitter power which lies in the direction finding of radiation sources by the algorithms which take into account the indenture and nonidentity of partial characteristics of the DAA receiving channels. Conclusions made with respect to (4) certainly are valid also for estimation (2). Applicable to it the said effects may be more noticeable due to the possibility of manipulating the nonidentity of not only amplitude, but also phase directivity characteristics.

The specific value of the advantage depends on the realized law of DC change along the angular coordinate. The result of mathematical simulation presented in Fig. 1 allows us to judge about it with the known degree of approximation. Figure 1 illustrates relationship (4) on condition that $K = 3$, $p_k = 1$ and the law of the DC change is described by the function $F_k = 100 - M + M \cdot \cos^2\left(E \cdot x + \pi \cdot \frac{k-z}{K}\right)$, where M is the DC indenture depth in percent. Along the horizontal axis the values of the generalizing coordinate x are plotted, along the vertical one - mean-square errors (MSE) of its measurements. The advantage in accuracy grows as DC indenture intensifies several-fold compared with identical characteristics (F_k = 100). The presence of the deviations of real values F_k from those used in relationship (3) may introduce substantial adjustments. For instance, as the results of simulating procedure (3), showed applicable to the version F_k considered here for the three-channel DAA and $E = 2$, the MSE critical values of unaccounted for fluctuations of the value M, whereby the said accuracy advantage is retained, do not exceed 3–5% (depending on the coordinate x). As the number of the channels and values E grows the requirements for the information validity with respect to the depth of modulation M becomes more rigorous.

In terms of generality of the statement, direction finding procedures based on the formation by phased summation of secondary channels deserves attention. Since with nonidentified DC of DAA receiving elements the specific form of the secondary channels depends on these nonidentities and is the function of the signal arrival direction, the output voltage of the rth synthesized channel may be written down in the form

$$\dot{U}_r = U_r^c + j\cdot U_r^s = \dot{a}\cdot \dot{F}_r(x) + \dot{n}_r =$$
$$= a^c\cdot F_r^c(x) - a^s\cdot F_r^s(x) + j\cdot(a^c\cdot F_r^s(x) + a^s\cdot F_r^c(x)) + \dot{n}_r,$$

where $\dot{F}_r(x) = F_r^c(x) + j\cdot F_r^s$ is the complex DC of a secondary channel.

Ignoring the correlation $\dot{U}_r$ in terms of noises $\dot{n}_r$ the logarithmic equivalent of the likelihood function will be written in the following way:

$$L_{\text{B}} = \ln C - \frac{1}{2}\cdot\sigma_{\text{n}}^{-2}\cdot\sum_{r=1}^{R}\;[\{U_r^c - a^c\cdot F_r^c(x) + a^s\cdot F_r^s(x)\}^2 +$$
$$+\{U_r^s - a^s\cdot F_r^{x}(x) - a^c\cdot F_r^c(x)\}^2].$$

Hence the Cramer–Rao bound in the fractions of the generalized angular coordinate x will take the form

$$\sigma_x^2 \geq \frac{\sigma_{\text{n}}^2}{a^2}\cdot\left\{\sum_{r=1}^{R}\;[(F_r^{c'}(x))^2 + (F_r^{s'}(x))^2] - W\cdot\left[\sum_{r=1}^{R}\;[F_r^{c^2}(x) + F_r^{s^2}(x)]\right]^{-1}\right\}^{-1}, \qquad (5)$$

where

$$W = \left\{\sum_{r=1}^{R}\;[F_r^c(x)\cdot F_r^{c'}(x) + F_r^s(x)\cdot F_r^{s'}(x)]\right\}^2 +$$
$$+\left\{\sum_{r=1}^{R}\;[F_r^{s'}(x)\cdot F_r^c(x) - F_r^{c'}(x)\cdot F_r^s(x)]\right\}^2;$$

σ_{n}^2 is the variance of noises in the quadrature component of the secondary channel response.

The accuracy estimate obtained corresponds to the measuring procedure boiling down to the maximization of the function:

$$L_{\text{MB}} = \left[\left\{\sum_{r=1}^{R}\;[U_r^c\cdot F_r^c(x) + U_r^s\cdot F_r^s(x)]\right\}^2 + \left\{\sum_{r=1}^{R}\;[U_r^s\cdot F_r^c(x) - U_r^c\cdot F_r^s(x)]\right\}^2\right]\times$$
$$\times\left\{\sum_{r=1}^{R}\;(F_r^{c^2}(x) + F_r^{s^2}(x))\right\}^{-1}, \qquad (6)$$

which differs from the known one according to [2] by the presence of the normalizing multiplier depending on the coordinate x.

Expression (5) does not take into account anomalous errors in measurements caused by the influence of diffraction maxima of function (6). When excluding $\text{Im}\,(\dot{F}_r(x))$ from consideration it is easily transforms into the estimate of accuracy of one-target direction finding oriented at real characteristics of the secondary channels:

$$\sigma_x^2 \geq \frac{\sigma_n^2}{a^2} \cdot \left\{ \sum_{r=1}^{R} (F_r'(x))^2 - \left\{ \sum_{r=1}^{R} F_r(x) \cdot F_r'(x) \right\}^2 \cdot \left[\sum_{r=1}^{R} F_r^2(x) \right]^{-1} \right\}^{-1}. \tag{7}$$

Hence it is not difficult to make a practically important conclusion that the greatest accuracy are given by the characteristics possessing the property:

$$\sum_{r=1}^{R} F_r(x) \cdot F_r'(x) = 0.$$

As is known, this category includes the functions of the form $\frac{\sin R \cdot (x - x_r)}{\sin (x - x_r)}$, which are widely practiced in DAA [2, 3].

Thus, the Cramer–Rao bounds considered make it possible to make a judgment on the accuracy of one-signal direction-finding procedures in the case of nonidentical DAA receiving channels.

In this case the synthesis of the set of nonidentical DC receiving channels minimizing MSE of direction finding becomes significant.

Similar estimates were obtained by the author also for the case of multi-reading measurements including those applicable to multi-signal direction-finding procedures of superresolution. It is essential that for coherent location the accuracy advantage at the expense of nonidentical DC may be obtained with respect to the output of the secondary channels by changing over time the weights of physical summation. In this case in estimates (5) and (7) the DC accumulation and their derivatives in terms of index r should be supplemented by summation with respect to S by time samples.

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