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SUPER-RAYLEIGH RESOLUTION OF NARROW-BAND PULSES IN TERMS OF DELAY TIME

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The paper considered algebraic methods of resolution of M signals, differences in delay time, which are fewer than the Rayleigh limit, based on approximation of the pulse envelope by the function $\sin^M \tilde{x}_M$.

There is a rather wide range of problems, when the required range resolution cannot be achieved by expanding the band of the sounding signal. Taking into account the opinion — rife among experts — on insufficient substantiation of using remote "super-resolution" and a preconception that "transition to any super-resolution is related to energy losses", [1] it requires serious adjustment.

The objective of the article is consideration of ranging procedures of super-Rayleigh resolution of narrow-band radio signals, whose realization results in energy losses only in the case of the deviation of an actual model of the signal from the designed one.

Among numerous procedures of "super-resolution" the processing methods based on noiseless idealization of the signal mixture and its analytical mapping by the polynomial, whose power equals the number of sources of signals, while the radicals are unambiguously associated with their parameters, represent an extensive group. [2, 4]. Let us use such an approach when synthesizing a measuring procedure at first for the two-signal situation of reception.

Taking into account argumentation [3], we approximate the narrow-band envelope by the function $\sin^2 x$, where given discrete mapping $x = \pi s / N$, s is the shift of the sth sample of ADC from the beginning of the signal, N is the pulse length by the base in digitization periods. Note, that for such model of the envelope the closest Gaussian analog will be the exponential curve

$$\exp\left\{-\frac{2.687 \cdot 4}{N^2}\left(s - \frac{1.5 \cdot N}{2}\right)^2\right\}. \tag{1}$$

It somewhat differs from the result shown in [5], which is explained by impossibility of itemization at the instance of publication [5] of the coefficients in argument (1) due to underdevelopment of computational technology.

When receiving two video pulses in complex mapping (with random quadrature components of amplitude and time of arrival) the number of unknown parameters of the signals without regard to noises equals six. Therefore, for presenting a normal system of equations making it possible to determine the desirable time delay of each of the signals, a trio of complex samples will be required. Their voltages within the existence limits of both pulses may be written down in the form

$$\dot{U}_1 = \dot{a}_1 \sin^2 \frac{\pi}{N} d_1 + \dot{a}_2 \sin^2 \frac{\pi}{N} d_2,$$

$$\dot{U}_2 = \dot{a}_1 \sin^2 \frac{\pi}{N} (d_1 + z_1) + \dot{a}_2 \sin^2 \frac{\pi}{N} (d_2 + z_1),$$

$$\dot{U}_3 = \dot{a}_1 \sin^2 \frac{\pi}{N}(d_1 + z_2) + \dot{a}_2 \sin^2 \frac{\pi}{N}(d_2 + z_2),$$

here d_m is the unknown shift of the first sample of the trio in digitization periods with respect to the beginning of the mth signal, z_1 and z_2 are intervals characterizing the shift of the second and third samples of the triad as regards the first.

To solve this system with respect to d_m let us conduct the separation of quadrature components of the amplitudes of the signals. Its result is a system of two equations, which by expansion of determinants into elements of the first columns is transformed to the form

$$\begin{cases} U_1^c - U_2^c x_1 + U_3^c x_2 = 0 \\ U_1^s - U_2^s x_1 + U_3^s x_2 = 0 \end{cases}, \quad (2)$$

where U_s^c, U_s^s are quadrature voltage components,

$$x_1 = \begin{vmatrix} \sin^2 \frac{\pi}{N} d_1 & \sin^2 \frac{\pi}{N} d_2 \\ \sin^2 \frac{\pi}{N}(d_1 + z_2) & \sin^2 \frac{\pi}{N}(d_2 + z_2) \end{vmatrix} : \mathrm{Det}, \quad (3)$$

$$x_2 = \begin{vmatrix} \sin^2 \frac{\pi}{N} d_1 & \sin^2 \frac{\pi}{N} d_2 \\ \sin^2 \frac{\pi}{N}(d_1 + z_1) & \sin^2 \frac{\pi}{N}(d_2 + z_1) \end{vmatrix} : \mathrm{Det},$$

$$\mathrm{Det} = \begin{vmatrix} \sin^2 \frac{\pi}{N}(d_1 + z_1) & \sin^2 \frac{\pi}{N}(d_2 + z_1) \\ \sin^2 \frac{\pi}{N}(d_1 + z_2) & \sin^2 \frac{\pi}{N}(d_2 + z_2) \end{vmatrix}. \quad (4)$$

Using the method of the least squares, and using the voltages of ADC samples it is possible from (2) to find estimates x_1 and x_2 applicable to zero power of noises. To this end one should minimize the functional composed in the general case by responses S of frequency filters (receiving channels of the antenna array):

$$F = \sum_{s=1}^{S} \left\{U_{1s}^c - U_{2s}^c x_1 + U_{3s}^c x_2\right\}^2 + \sum_{s=1}^{S} \left\{U_{1s}^s - U_{2s}^s x_1 + U_{3s}^s x_2\right\}^2. \quad (5)$$

Multichannel nature of processing in the given case makes it possible to raise the accuracy of range measurements in unfavorable reception situation in terms of the differences of the initial signal phases.

As a result of minimization [5] it is not difficult to determine

$$x_1 = \left\{\sum_{s=1}^{S} U_{3s}^2 \sum_{s=1}^{S} U_{12s} - \sum_{s=1}^{S} U_{13s} \sum_{s=1}^{S} U_{32s}\right\} : \mathrm{Det}, \quad (6)$$

$$x_2 = \left\{-\sum_{s=1}^{S} U_{2s}^2 \sum_{s=1}^{S} U_{13s} + \sum_{s=1}^{S} U_{23s} \sum_{s=1}^{S} U_{12s}\right\} : \mathrm{Det}, \quad (7)$$

$$\mathrm{Det} = \sum_{s=1}^{S} U_{2s}^2 \sum_{s=1}^{S} U_{3s}^2 - \left\{\sum_{s=1}^{S} U_{23s}\right\}^2,$$

$$U_{ns}^2 = U_{ns}^{c\,2} + U_{ns}^{s\,2}, \; U_{nms} = U_{ns}^c U_{ms}^c + U_{ns}^s U_{ms}^s.$$

On the other hand, the values x_1 and x_2 according to (3) may be correlated to unknown d_1 and d_2. Omitting cumbersome calculations from determinants (3) and (4) we will obtain:

$$P = \frac{x_1 \sin^2 \frac{\pi}{N} z_1 - x_2 \sin^2 \frac{\pi}{N} z_2}{x_1 \cos^2 \frac{\pi}{N} z_1 - x_2 \cos^2 \frac{\pi}{N} z_2 - 1}, \tag{8}$$

$$S = \frac{x_2 \sin^2 2 \frac{\pi}{N} z_2 - x_1 \sin^2 2 \frac{\pi}{N} z_1}{x_1 \cos^2 \frac{\pi}{N} z_1 - x_2 \cos^2 \frac{\pi}{N} z_2 - 1},$$

where $P = \tan \frac{\pi}{N} d_1 \cdot \tan \frac{\pi}{N} d_2$, $S = \tan \frac{\pi}{N} d_1 + \tan \frac{\pi}{N} d_2$.

Further substitution of the values x_1 and x_2 from (6) and (7) into relationship (8) makes it possible to reduce the search for the estimates $\tan \frac{\pi}{N} d_1$ and $\tan \frac{\pi}{N} d_2$ to the solution of the square equation

$$\tan^2 \frac{\pi}{N} d_m - S \tan \frac{\pi}{N} d_m + P = 0.$$

Hence

$$d_{1,2} = \frac{N}{\pi} \arctan\left(-\frac{S}{2} \pm \sqrt{\frac{S^2}{4} - P}\right).$$

It should be noted that the result at the accuracy of up to an argument of tangents coincides with the solution of the two-signal problem of spectral estimation by the output voltages of the FFT-filters. Similarly to [6], the roots of the square equation are advisable to be calculated by means of a permanent storage using as input addresses the coefficients P and S outputting the results in the form of sought-for estimates d_1 and d_2.

As for a large number of sources, then their resolution in time delay within the framework of the envelope $\sin^2 x$ is possible only based on iteration procedures. Therefore, for reducing the ranging problem to the solution of the algebraic equation of power M in the general case it is proposed to use as an envelope analytical model the function $\sin^M (\tilde{x}_M)$, argument whose index of a power would vary depending on the number of M sources. The choice of the value $\tilde{x}_M$ is called for to ensure optimal approximation of Gaussian signal (1) by the trigonometric equivalent with any order of the model. Corresponding approximations of exponent (1) by the power functions of sines were established by the programmed plotting of the envelopes and visual selection of the most convenient of them. The enumeration of the best approximation of this type, limited $M = 10$, are shown in Table 1. It is noteworthy that as M grows the quality of approximation improves.

One of the generalized versions of the multi-signal measuring procedure may be reduced to the solution of the algebraic equation composed of $M + 1$ samples of complex voltages M of the filters (channels):

$$\begin{vmatrix} \sin^M \tilde{d}_M & \sin^M (\tilde{d}_M + \tilde{z}_{M1}) & \sin^M (\tilde{d}_M + \tilde{z}_{M2}) & \cdots & \sin^M (\tilde{d}_M + \tilde{z}_{MM}) \\ \dot{U}_{1,1} & \dot{U}_{2,1} & \dot{U}_{3,1} & & \dot{U}_{M+1,1} \\ \dot{U}_{1,2} & \dot{U}_{2,2} & \dot{U}_{3,2} & \cdots & \dot{U}_{M+1,2} \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot \\ \dot{U}_{1,M} & \dot{U}_{2,M} & \dot{U}_{3,M} & \cdots & \dot{U}_{M+1,M} \end{vmatrix} = 0. \tag{9}$$

Here $\tilde{d}_M$ denotes the argument of the function sin reduced in Table 1 for the power M with the only difference being instead of the sample number s the variable d_m is used corresponding to the shift of the first sample of measuring sampling with respect to the beginning of the mth signal. Similarly, $\tilde{z}_{M\,n-1}$ should be understood as an interval between the nth and first samples of the sampling digitization periods multiplied by the coefficient T_M given the variable s in the same argument of the sine in Table 1. In this case it is assumed that in all samples of the sampling process there are voltages M of the signals. In order to go to tangents, it is enough in determinant (9) to divide the first row by $\cos^M \tilde{d}_M$.

Table 1

M	Model of the envelope $\sin^M \{T_M s + H_M\}$
3	$\sin^3 \left\{\frac{\pi}{1.21\,N} s + 0.27\right\}$
4	$\sin^4 \left\{\frac{\pi}{1.4\,N} s + 0.45\right\}$
5	$\sin^5 \left\{\frac{\pi}{1.54\,N} s + 0.55\right\}$
6	$\sin^6 \left\{\frac{\pi}{1.687\,N} s + 0.64\right\}$
7	$\sin^7 \left\{\frac{\pi}{1.8\,N} s + 0.7\right\}$
8	$\sin^8 \left\{\frac{\pi}{1.928\,N} s + 0.755\right\}$
9	$\sin^9 \left\{\frac{\pi}{2.0527\,N} s + 0.8085\right\}$
10	$\sin^{10} \left\{\frac{\pi}{2.16\,N} s + 0.845\right\}$

Considering each of the quadratures separately, it is not difficult to obtain a system of two determinants of the M + 1th order corresponding to cosine and sine components of the voltages. Such transformation (9) is useful for determining $\tilde{d}_M$ using the method of the least squares. In the case $M = 2$ we obtain the functional of errors (5).

It is important, that quadrature components may be used also when forming determinant (9) substituting this or other of the quadratures in place of complex voltage samples. This technique makes it possible to reduce the number of frequency filters (channels) required for the measuring procedure. For instance, $M + 1$ samplings given even powers M are sufficient to be composed with responses $M/2$ of frequency filters, while in the case odd powers — $(M + 1)/2$. In the last case one of the quadratures of any filter is discarded. It is necessary to take into account that similar saving in the volume of data is accompanied by the growth of accuracy sensitivity, in terms of estimating signal time delay, to unfavorable relationships of their initial phases.

The approach described until now was discussed applicable to video pulses. Its generalization to radio signal processing implies, similarly to [2], phase shift correction of all ADC samples involved except the first according to the procedure:

$$\tilde{U}_s^c = U_s^c \cos p_s + U_s^s \sin p_s; \quad \tilde{U}_s^s = U_s^s \cos p_s - U_s^c \sin p_s, \qquad (10)$$

where $p_s = \omega \, \Delta \, t \, z_s$, z_s is the time shift of the sth sample with respect to the first in the sampling processed in the periods of ADC digitization.

In multisignal situation such technique is justified given minor differences in the frequency of oscillations filling radio pulses. At the same time, inadequacy of phasing by its emergence may be quite well referred to the equivalent worsening of the signal-to-noise ratio.

The above phase adjustment is necessary also in that case, when M of signals is not present in all samples of the sampling. In this case as a result of such operation, the difference of the initial phases of the signals is altered artificially, which may be useful given their unfavorable relationships, the general form of the complex amplitude of the mth signal, subject to separation after additional phasing (10), may be represented in the following way:

$$\tilde{a}_m = a_m \exp \{ j [\varphi_m + \omega_m \Delta t d_m + \omega_m \Delta t z_{ms} - \omega_m \Delta t z_{1s}]\},$$

where z_{ms} is the shift of the sth ADC sample with respect to the origin of the mth pulse d_m in digitization periods Δt; z_{1s} is the shift of the ADC sth sample with respect to the origin of the measuring sampling (d_1); a_m, φ_m, ω_m is the amplitude, the initial phase, and filling frequency of the mth radio pulse (in each sample of the array processed all M signals are present, then $z_{ms} = z_{1s}$).

To check the hypotheses with respect to the number of sources and the presence or absence of their signals on the sampling edges it is advisable to use an approach similar to [2], maximizing, using the obtained estimates of the time position of the pulses, the functional:

$$F_M = \begin{vmatrix} 0 & \dot{W}_1^*(M) & \dots & \dot{W}_M^*(M) \\ \dot{W}_1(M) & Q_{11}(M) & \dots & Q_{1M}(M) \\ \dots & \dots & \dots & \dots \\ \dot{W}_M(M) & Q_{M1}(M) & \dots & Q_{MM}(M) \end{vmatrix} \cdot \begin{vmatrix} Q_{11}(M) & Q_{12}(M) & \dots & Q_{1M}(M) \\ Q_{21}(M) & Q_{22}(M) & \dots & Q_{2M}(M) \\ \dots & \dots & \dots & \dots \\ Q_{M1}(M) & Q_{M2}(M) & \dots & Q_{MM}(M) \end{vmatrix}^{-1};$$

$$\dot{W}_n(M) = \sum_{s=r_n}^{r_n+N} (U_s^c + j U_s^s) K_{sn}(M); \quad Q_{pn}(M) = Q_{np}(M) = \sum_{s=r_p}^{r_n+N} K_{sp}(M) K_{sn}(M);$$

s is the current number of the digitization interval, r_n is the number of the first of the digitization intervals within the existence limits of the nth signal,

$$K_{sn}(M) = \begin{cases} \sin^M (T_M (s - r_n) + H_M) & \text{at } r_n \le s \le r_n + N \\ 0 & \text{at } s < r_n \text{ and } s > r_n + N \end{cases},$$

T_M and H_M are coefficients from Table 1, N is the length of the pulses.

What is new in the given case is the fact that for each value M the proper model of the envelope is used. If it is advisable to have one and the same approximation of the signal forms for the arbitrary M, then to the detriment of the compact nature of computational operations one may confine himself to the model corresponding to the maximum of the expected number of sources.

The reduction of the problem of estimating the time position of the pulses to the solution of the algebraic equation makes it possible to substantially reduce the computational volume of operations. The possibility of using the modernized Lobachevskii method for highly accurate determination of roots makes the algebraic approach especially enticing [7].

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