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MULTIFREQUENCY OPERATION OF COMMUNICATION CHANNELS BASED ON SUPER-RAYLEIGH RESOLUTION OF SIGNALS

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The paper considers several approaches to frequency-division multiplexing of narrow-band signals based on the super-Rayleigh resolution in frequency.

The majority of known approaches to the improvement of transmission capacity of communication channels consist in expanding their spectral band. In turn it leads to a number of problems such as the electromagnetic compatibility of heterogeneous equipment and the deficiency of frequency resources in the most extensively used ranges of the electromagnetic spectrum. This situation can be improved if the processing of communication signals is performed based on their super-Rayleigh resolution.

The purpose of this work is to consider the communication channel multiplexing in the case of discrete multi-frequency coding of information. The reference point of our investigation is the known method of orthogonal frequency discrete modulation (OFDM), which in the last few years has been widely used, for example, in ADSL systems [1] and in digital television (DVB-T).

The processing principle suggested below has much in common with OFDM (Fig. 1) and differs in a denser packing of the carrying signals in the transmitter (Fig. 2) oriented to their further super-Rayleigh resolution, which must not reduce the reliability of the information.

Provided that the frequency coding is accompanied by amplitude-phase modulation of the carrier frequencies, the estimates of quadrature components of signals in the receiver can be obtained from the voltages of frequency filters synthesized by the fast Fourier transform (FFT). In the case of deterministic interpretation of the signal mixture in the absence of Doppler's shifts of frequency, the respective relationships have the form [2]

$$\hat{a}_m^{c(s)} = \frac{\det_m^{c(s)}}{S \cdot \det}; m = 1, 2, \ldots, M, \tag{1}$$

where S is the dimensionality (number of points) of the FFT operation; $\det_m^{c(s)}$ is a partial determinant obtained from the determinant

$$\det = \begin{vmatrix} f_1(w_1) & f_1(w_2) & \cdots & f_1(w_M) \\ f_2(w_1) & f_2(w_2) & \cdots & f_2(w_M) \\ \vdots & \vdots & \cdots & \vdots \\ f_M(w_1) & f_M(w_2) & \cdots & f_M(w_M) \end{vmatrix}$$

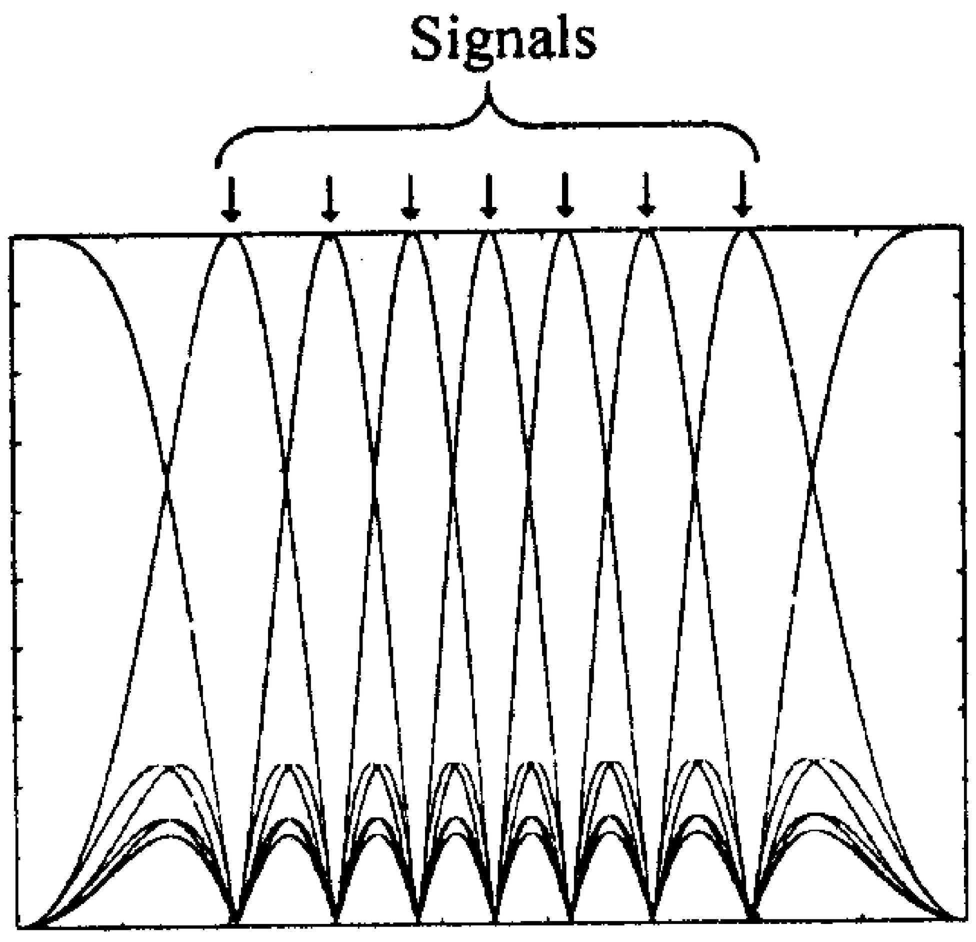

Fig. 1

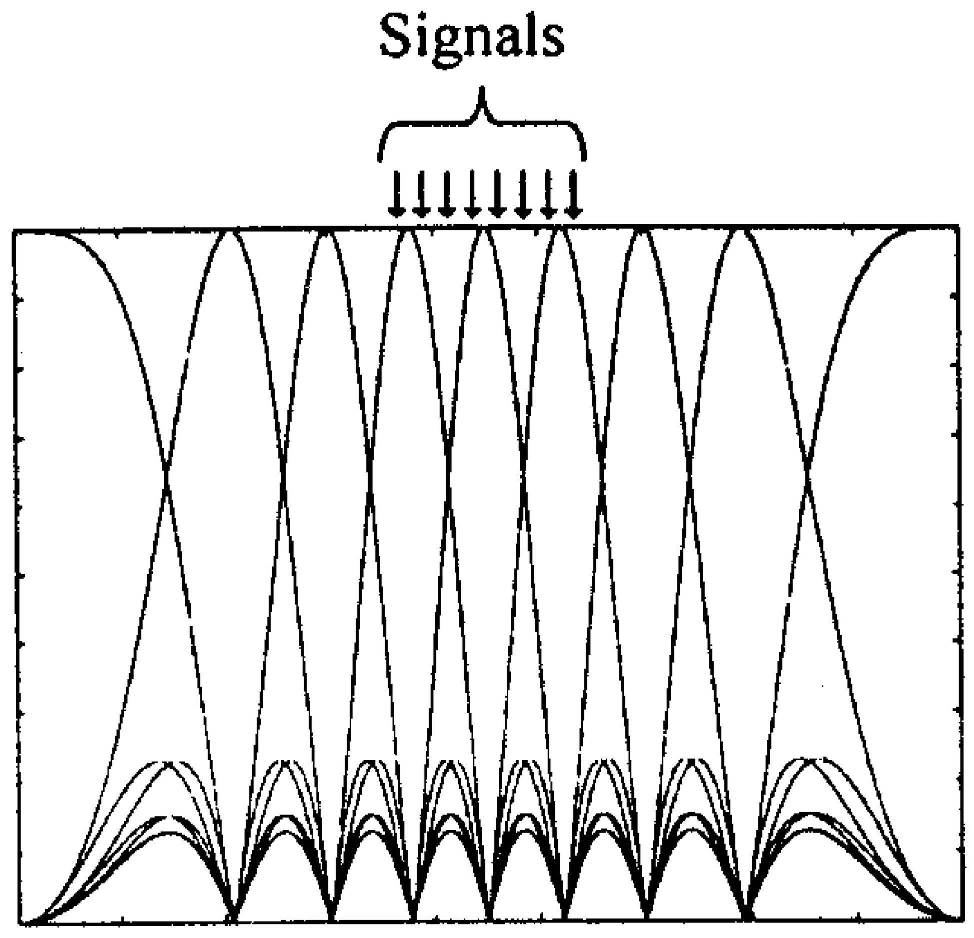

Fig. 2

by replacing the respective column by the vector of free terms $[B^{c(s)}] = [U_1^{c(s)}\, U_2^{c(s)} \cdots U_M^{c(s)}]^{\mathrm{T}}$, where $U_j^{c(s)}$ are quadrature components of the complex response of the jth FFT-filter; $f_j(w_m) = \sin S\left[j \cdot \frac{\pi}{S} - w_m\right] / \sin\left[j \cdot \frac{\pi}{S} - w_m\right]$ is the value of the amplitude-frequency response (AFR) of the jth frequency filter synthesized by means of FFT; and w_j, w_k and w_m are known carrier frequencies among the totality of the prescribed frequencies expressed in segments of the whole width of the AFR main "lobe" of the FFT-filter.

Relation (1) was derived after solving a system of equations composed by the response voltages of the frequency filter without taking noise into account:

$$U_j^{c(s)} = \sum_{m=1}^{M} a_m f_j(w_m),\ j = 1, \ldots, M.$$

To make the treatment of real-valued signals easier, their analog-to-digital conversion has to be performed with the period of sampling multiple to an odd number of quarters of the period of the information packet central frequency. Then the partition of $2S$ samples, obtained from ADC ($2S > M$), into those with even and odd ordinal numbers makes it possible to easily form the quadrature components of voltages for subsequent operation of the S-point FFT.

In the most demanding applications, generation of the quadratures from real-valued signals can be performed by means of the discrete Hilbert transformation, which is realized in the sliding window conditions over a given number of ADC samples dictated by the Hilbert filter order. In this case the overall number of ADC samples formed on the measurement sample interval must exceed the dimensionality of the FFT procedure by the doubled interval of the transient process of the Hilbert filtering [3].

To make the most use of the signal energy and to estimate optimally the amplitude components by the least-square method, the denominator of relation (1) must contain the determinant

$$\det = \begin{vmatrix} S & f_{12} & \cdots & f_{1M} \\ f_{12} & S & \cdots & f_{2M} \\ \vdots & \vdots & \cdots & \vdots \\ f_{1M} & f_{2M} & \cdots & S \end{vmatrix}, \qquad (2)$$

while the numerator of (1) is a partial determinant $\det_m^{c(s)}$ obtained from (2) by replacing the respective column by the vector of free terms [2]

$$[B^{c(s)}] = \left[\sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_1) \sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_2) \ldots \sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_M) \right]^{\mathrm{T}}$$

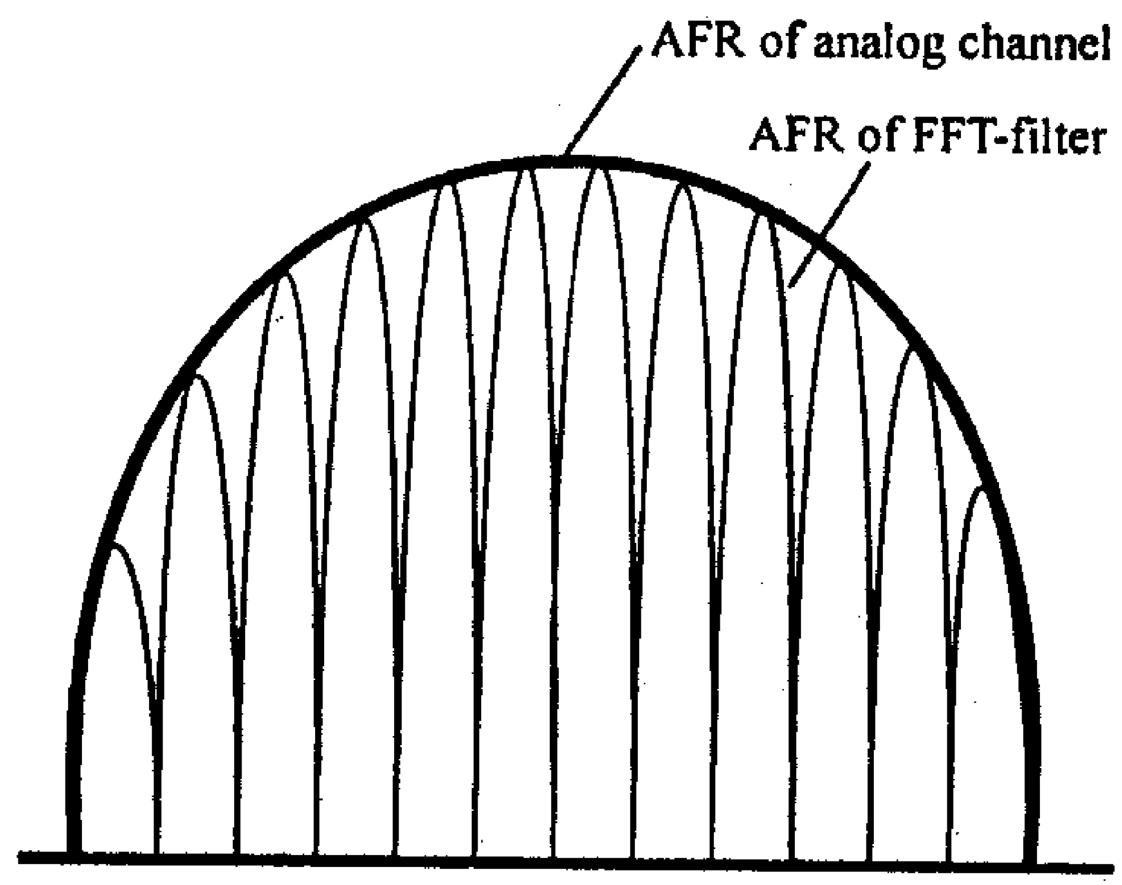

Fig. 3

where $U_j^{c(s)}$ are quadrature components of the complex response of the jth FFT-filter,

$$f_j(w_m) = \sin S \cdot \left[j\frac{\pi}{S} - w_m \right] / \sin\left[j\frac{\pi}{S} - w_m \right]$$

is the value of AFR of frequency filters synthesized by FFT, and $f_{jk} = \sin S \cdot (w_j - w_k) / \sin(w_j - w_k)$.

The potential accuracy of measurement of quadrature components comprising the signals in a multifrequency packet depends first of all on the signal-to-noise ratio at the outputs of the frequency channels synthesized, and second on the spacing between the carrier frequencies in the spectral domain. The increase in traffic capacity may take the form of a denser arrangement of carriers, or compression of the accumulation interval used for creating FFT-filters, which leads to an expansion of their passband.

An example illustrating the operability of the procedures suggested represents an approbation of a similar set of operations over a multifrequency packet [4] as applied to the measurement of amplitude-frequency responses of a radio system.

The practical implementation of the approach suggested is as follows. In the receiver of information messages we use a digital signal processor or a programmable matrix of logic elements (produced by the "Xilinx" company, for instance) involved into processing of ADC samples in conformity with the above formulas. For ADC one can take high-speed converters described in [5]. At the transmitting side, to generate the multisignal mixture, it is appropriate to employ a digital signal processor and a DAC produced, for example, by the "Analog Devices" company.

Derivation of relationships (1) and (2) was performed without taking into account the frequency-selective properties of a real communication line, whose narrow-bandness may result in regular errors in the measurement of signals' quadrature components. The errors arise from disparity in final responses of FFT-filters because of nonuniformity of the transfer factor $K(\omega)$ of the analog channel in the frequency domain (Fig. 3). This problem is aggravated by the fact that to realize relationships (1) and (2) we have to use a set of frequency filters, whose number must correspond to the dimension of the set of carrier frequencies used in communication. As a result, at large samples of the frequency channels, a part of FFT-filters fall outside the main passband of the whole analog channel, located at the "slopes" of its AFR. Thus, the procedures of message decoding (1) and (2) must be preceded by restoration of levels of the carrier frequencies of the initial packet. It can be made, for example, by weighting the quadrature components of voltages at FFT output, i.e., multiplying them by the quantities, which are inverse to the analog channel transfer factor $K(\omega)$ in the points of maximum of the responses of the respective filters. This weighting has to be performed with by taking into account the proper location of the signal packet frequencies around the maximum of AFR of the analog channel – to "freeze" the responses of frequency filters at the ends of the corresponding spectral segment, and to lower their level in the center of the processing band. It allows us to avoid, in the process of weighting, a deterioration of the signal-to-noise ratio in the responses of the bordering filters.

A stricter account of the communication line AFR implies that estimates of quadrature components of amplitudes are formed not with the aid of (2) but using the expressions

$$\det = \begin{vmatrix} \sum_{j=0}^{S-1} f_j^2(w_1) & \sum_{j=0}^{S-1} f_j(w_1) f_j(w_2) & \cdots & \sum_{j=0}^{S-1} f_j(w_1) f_j(w_M) \\ \sum_{j=0}^{S-1} f_j(w_1) f_j(w_2) & \sum_{j=0}^{S-1} f_j^2(w_2) & \cdots & \sum_{j=0}^{S-1} f_j(w_2) f_j(w_M) \\ \vdots & \vdots & \cdots & \vdots \\ \sum_{j=0}^{S-1} f_j(w_1) f_j(w_M) & \sum_{j=0}^{S-1} f_j(w_2) f_j(w_M) & \cdots & \sum_{j=0}^{S-1} f_j^2(w_M) \end{vmatrix}, \tag{3}$$

$$[B^{c(s)}] = \left[\sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_1) \sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_2) \ldots \sum_{j=0}^{S-1} U_j^{c(s)} \cdot f_j(w_M) \right]^{\mathrm{T}},$$

where, depending on the assumptions adopted, $f_j(w_m)$ may represent the products

$$K\left(j\cdot\frac{\pi}{S}\right)\sin S\cdot\left[j\cdot\frac{\pi}{S}-w_m\right]/\sin\left[j\cdot\frac{\pi}{S}-w_m\right] \quad \text{or} \quad K(w_m)\sin S\cdot\left[j\cdot\frac{\pi}{S}-w_m\right]/\sin\left[j\cdot\frac{\pi}{S}-w_m\right].$$

Here $K\left(j\cdot\frac{\pi}{S}\right)$ or $K(w_m)$ correspond to the values of the normalized AFR of the analog communication line in the maximum of the jth frequency filter and at a prescribed frequency w_m expressed in segments of the width of FFT-filter response.

The engagement into communication, in the approach suggested, must be preceded by prompt adaptation of the coding amplitude levels to the interference situation and to noise power in the line. The peculiar feature of the adequate processing of signals at the reception side consists in measurements, with the use of relations (1)–(3), of amplitude components of the interference at every used carrier frequency, when the transmitter is turned off. The results of this testing must be used for adaptation of the methods of signal modulation in the multifrequency packet. For example, at some frequencies we may implement the procedure of 64QAM-coding, while at those subject to noise perturbations — 4PSK or even BPSK.

In order to take the reflection effect in the communication channel into account, it is expedient to estimate the levels of the respective interference by a test signal of the transmitter, whose quadrature components have fixed values a priori known to the reception side. The interference situation parameters are evaluated through a comparison between the carrier amplitudes measured with the aid of (1)-(3) and some standard values.

Analysis of potentialities of the frequency channeling with the aid of super-Rayleigh resolution can be performed based on calculating the measurement error variances of amplitude components at the lower Cramer-Rao boundary. The general form of the Fisher matrix employed at the establishment of the lower Cramer-Rao boundary is defined by

$$I = F^{\mathrm{T}} F / \sigma^2 \tag{4}$$

where the elements of the matrix block $F^T F$ are identical to elements of the determinant det in (2) and (3) while σ^2 is the variance of noise in the quadrature component of the FFT-filter response.

For non-correlated samples $\sigma^2 = S\cdot\sigma^2_{\mathrm{ADC}}$, where S is the FFT dimensionality and σ^2_{ADC} is the noise variance in the ADC sample. To avoid the division by zero in the expressions for frequency filter responses, their calculation in (4) can be performed with the use of a well-known identity

$$f_r(x) = 2\sum_{n=1}^{N/2} \cos\left[2(x-r)\cdot\frac{\pi}{N}\left(n-\frac{N+1}{2}\right)\right].$$

Investigation of potentialities of frequency-division multiplexing of communication channels based on the suggested approach was carried out by selective calculation of variances of nonbiased estimates of amplitudes as diagonal elements of (4) for the case of three, four, and five signals. To make the task easier, we analyzed the strictly amplitude method of multilevel coding, without using "dequadratured" signals. It was assumed that all carriers had the zero initial phase and

Table 1

Number of FFT points, *S*	Methods using super-Rayleigh resolution of signals					OFDM		
	2-frequency packet			4-frequency packet		Without recalculation of noise variance	For recalculated noise variance	
							With regard for doubling the frequency band	With regard for 4-fold expansion of frequency band
	Frequency spacing of signals in fractions of FFT-filter width							
	0.5	**0.25**	**0.125**	**0.5**	**0.25**	**1.0**	**1.0**	**1.0**
16	0.324	0.576	1.116	1.072	9.984	0.25	0.3535	0.5
32	0.232	0.407	0.786	0.747	6.92	0.1767	0.25	0.3535
64	0.16	0.288	0.56	0.528	4.872	0.125	0.1767	0.25
128	0.113	0.204	0.396	0.373	3.439	0.0884	0.125	0.1767
256	0.081	0.144	0.272	0.256	2.432	0.0625	0.0884	0.125
512	0.057	0.101	0.197	0.186	1.72	0.0442	0.0625	0.0884
1024	0.0405	0.072	0.139	0.131	1.216	0.03125	0.0442	0.0625

were shifted from one another by equal frequency increments. The noise variance, recalculated to ADC output, was taken to be equal to unity, and the quantization noise was not considered.

Table 1 shows the calculation results for mean-square errors of amplitude measurements in ADC samples for the case of synthesis of *S*-frequency filters and of the interval between the harmonics represented in fractions of the FFT-filter passband. The table also contains the mean-square error for amplitude measurements by the OFDM method, obtained by calculations from the respective version of (4). Of importance is the fact that in the case of OFDM-coding the errors of the amplitude measurement do not depend on the number of carriers used while at super-resolution this dependence is noticeable.

Obviously, an increase in spacing between frequencies of the elementary channels requires the expansion of the passband of the whole analog channel, which in turn leads to increase in the noise power defined as $\sigma_N^2 = N_0 \cdot \Delta f$, where Δf is the frequency band of the group-containing channel, and N_0 is the noise spectral density. Assuming that the AFR width of the analog channel is strictly equal to the band occupied by the frequency channels, we can perform a consistent comparison of accuracy of the OFDM method with the method suggested in this work. To do this, we have to recalculate the mean-square errors of amplitude measurement obtained for OFDM taking into account the necessity in expanding the analog preselector band when passing from the super-resolution methods to the OFDM procedure. The results of this recalculation are presented in two right-hand columns of Table 1.

The column of mean-square errors of amplitude measurement using the OFDM method in the absence of noise variance recalculation points to the fact that if we use the super-resolution procedures, with the analog channel band equal to signals' band at OFDM-modulation, the super-resolution leads to a deterioration in the accuracy of decoding messages. This effect becomes more pronounced as the number of the participant carriers increases.

Under the condition of strict coincidence of passbands of the analog preselector and of the frequency channels, the two-frequency method of coding with super-resolution, when the carriers are spaced by a half-band of the FFT-filter, makes it possible to obtain more precise estimates of amplitudes than OFDM. It can be easily confirmed if we compare the values of the second in the left and second in the right columns in Table 1. This advantage of the two-frequency method of super-resolution over OFDM, in the event of coincidence of the preselector's band with the band of frequency channels, holds until the distance between the carriers reaches a quarter of the FFT-filter passband.

Further compression of the frequency channels results in a multiple increase in mean-square errors of amplitude measurements (see the fourth in the right column of Table 1). It should be noted, however, that at the ultimate selection of the method we have to consider the requirements imposed on the accuracy of the measurement. In practical situations, when we deal with the class of QAM-coding, most widely used (in terms of maximum attainable packing of bits in a single bod) is the 256QAM modulation making it possible to break the signal amplitude into 8 levels along each of the quadrature components. It is known that for a 13-digit ADC the overall number of signal mixture samples is 8192 or, with the sign position taken into account, 4096 of either polarity. If we divide this number by the number of levels equal to 8, it turns out that each amplitude level corresponds to 512 steps of ADC. To ensure the reliable decoding of information, we may assume that the interval between the points in the signal constellation (i.e., between the amplitude levels) must be $6\sigma_a$, where σ_a is MSD of the error of signal amplitude measurement, which is invariant with the amplitude level nominal value. With all this taken into account, the permissible value $\sigma_a = 512/6 = 85$ steps. Under such restrictions to measurement accuracy, as can be seen from Table 1, for a group of four carriers, even beginning from the 16-point length of the measurement sampling, the realization of a super-Rayleigh resolution makes it possible to guarantee a reliable communication when a signal's frequencies are separated from one another by one eighth of the synthesized FFT-filter. The overall band of the frequency packet in this case is limited by a half-band of the FFT-filter, which corresponds to the 8-fold frequency multiplexing in comparison to the traditional OFDM. Thus, in the example under consideration, the unjustifiably high accuracy of amplitude measurement due to OFDM, which is attained because of expansion of the frequency packet band, exceeds the required one considerably. The measurement accuracy in OFDM is not adequate for the accurate formation of information messages at the transmission side, where a step (difference between two adjacent values) of the digit-to-analog converter, as in ADC of the receiver, usually corresponds to the mean-square noise value. Thus, it seems possible, within the framework of QAM-modulation and the like (despite the deterioration of the amplitude measurement accuracy), to use the super-resolution of signals, providing a better compactness of communication channels in the spectral domain.

As a whole, an increase in the rate of channeling or its retention at a given level when the number of carriers increases, can be attained by increasing the word-length of ADC up to 16, 18, and even 24 digits, and by increasing the duration of measurement samples during the quantization periods. For example, for a millisecond interval of existence of legitimate signal in the digital television standard DVB-T, when an ADC with conversion frequency 100 MHz is used, we can generate 100 thousand ADC readings. By the way, for signal samples of such a long duration, a more serious limitation is the necessity in computations on the real-time basis.

After investigating the allowable limits of the frequency-division multiplexing, it has been established that the maximum number of carriers in a single filter has to be no more than five, otherwise the realization of acceptable accuracy requires a considerable signal-to-noise ratio, or a rather long sample (at least 64 readings). If using 8 tonal components, we may speak of the possibility of fourfold channeling of the information flow. At a larger number of harmonic carriers, their arrangement in the frequency domain shall be carried out in the form of four-signal blocks, with a spacing between these "quartets" in frequency, for example, at least two times longer than the interval between the signal carriers within the block. Then for a 16-component packet the maximum gain in traffic capacity will be limited by the triple value. According to the method suggested, we can analyze the marginal capacities of spectral multiplexing for more representative frequency sets.

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