ISSN 0735-2727

Radioelectronics and Communica tions Systems

(Iz. VUZ: Radioelektronika)

Volume 39
Number 5
1996

Allerton Press, Inc.
New York

SYNTHESIS OF ALGORITHMS FOR MEASUREMENT OF RANGE TO M SOURCES WITH THE USE OF ADDITIONAL GATING OF THE ADC READINGS

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Izvestiya VUZ. Radioelektronika,
Vol. 39, No. 5, pp. 55-62, 1996

UDC 621.396.967

The paper presents several optimal (in terms of the maximum likelihood method) procedures for the super-Rayleigh range resolution of M sources. The procedures are based on additional gating of the ADC readings by means of their coordinated summation within the preset and nonoverlapping time intervals.

One of the approaches to improve digital processing of signals consists in going to their analog-to-digital conversion (ADC) at earlier stages using the highest possible quantization frequency. The problem of matching the data transmission rate to the computer speed may be solved by the use of additional gating of the ADC readings in the form of the so-called partial summation [1] or accumulation with reset [2]. The gist of the method is as follows: a series of the ADC readings is transformed into a single integral reading which is rigidly related in time to the pulse graticule of the ADC clock period. This type of processing permits us to decimate the information without energy losses, to decrease the correlation of signals due to their enlarged representation, and to realize the procedures of the super-Rayleigh resolution of the sources in terms of range. The purpose of the present paper is the synthesis of the appropriate measurement algorithms.

To simplify the mathematics assume that the radio pulses to be processed have invariable duration, a nonmodulated carrier, an arbitrary but known envelope, and a specified filling frequency. Assume also that the real signals are converted by ADC into complex readings of the signal mixture voltages $\dot{U}_s$ through the time-sliding discrete Gilbert's transform. The subsequent summation with reset for N of these readings, unlike [1,2], may be performed in the form

$$U_t^c = \sum_{s=1}^{N} (U_s^c \cos P_s + U_s^s \sin P_s), \; U_t^s = \sum_{s=1}^{N} (U_s^s \cos P_s - U_s^c \sin P_s)$$

where $P_s = \omega \, \Delta t \cdot (s - 1)$, $U_s^c = \mathrm{Re}\,\{\dot{U}_s\}$, $U_s^s = \mathrm{Im}\,\{\dot{U}_s\}$, s is the ordinal number of the complex reading; ω is the filling frequency of the radio pulse; and Δt is the period of signal quantization such that

$$\omega \, \Delta t \cdot N = k \cdot 2 \cdot \pi, \; k = 1, 2, \ldots .$$

In the case when signal duration exceeds the time N of the accumulation (gating) interval, the reception is concurrent to the generation of a voltage set depending on the set of the M pulses:

$$\dot{U}_t = \sum_{m=1}^{M} \dot{a}_m \cdot S_t(z_m) + \dot{n}_t$$

where $\dot{U}_t = U_t^c + j \cdot U_t^s$ is the signal mixture complex voltage in the tth summarized reading (the tth gating); $\dot{a}_m = a_m^c + j \cdot a_m^s$ is the complex amplitude of the mth signal; M is the number of gated pulses;

$$S_t(z_m) = \begin{cases} \sum_{S=0}^{z_m - 1} K_m(S) \text{ at } t = t_{nm} \\ \sum_{S = z_m + N \cdot (t - 1 - t_{nm})}^{z_m + N \cdot (t - t_{nm}) - 1} K_m(S) \text{ at } t_{nm} < t \le t_{lm} \\ 0 \text{ at } t < t_{nm} \text{ and } t > t_{lm}, \end{cases} \quad (1)$$

t_{nm} is the ordinal number of the first gating among those containing the mth signal; z_m is the unknown bias of the mth pulse in the ADC readings with respect to the leading edge of the gating having the number $t = t_{nm} + 1$; N is the gating duration in the ADC readings; t_{lm} is the ordinal number of the last gating among those containing the mth pulse; $K_m(S)$ is the mth pulse discrete envelope referred to its peak value; and $\dot{n}_t$ is the noise complex value.

Having assumed that the noise quadrature components have the normal distribution, are noncorrelated, and possess the same time-independent variance, let us employ the maximum likelihood method for our measurement procedure synthesis. Then, up to a constant factor C for the set T of summarized readings of the gatings containing at least one pulse voltage, let us write the classical likelihood function

$$L = C \exp\left(-\frac{F}{2\sigma^2}\right) \quad (2)$$

where

$$F = \sum_{t=1}^{T} \left[\left\{ U_t^c - \sum_{m=1}^{M} a_m^c \cdot S_t(z_m) \right\}^2 + \left\{ U_t^s - \sum_{m=1}^{M} a_m^s \cdot S_t(z_m) \right\}^2 \right] \quad (3)$$

and σ^2 is variance of the summarized noise in the quadrature component of the gating response.

On the strength of one-to-one conformity between $U_t^{c(s)}$, $a_m^{c(s)}$, and z_m, the required range estimates may be determined through minimization of (3). The necessary condition for the problem decision may be formulated as follows: the number of gatings T and the number of sources M must obey the $2T \ge 3M$ relation. To measure the unknown delay times z_m of signals as was shown in [3], let us pass to another version (2) of the likelihood function. Upon differentiating (3) with respect to a_n^c and a_n^s, and denoting $W_n = \sum_{t=t_{Hn}}^{t_{Kn}} \dot{U}_t \cdot S_t(z_n)$, we may use Cramer's rule for the normal equation systems to find easily the required estimates of the amplitude components:

$$a_m^{c(s)} = \det D_{Mm}^{c(s)} / \det D_M \quad (4)$$

where $\det D_M$ is a symmetric determinant whose principal diagonal contains elements $\sum_{t=t_{Hn}}^{t_{Km}} S_t^2(z_m)$ while other entries represent the sums

$$\sum_{t=t_{Hn}}^{t_{Km}} S_t(z_m) S_t(z_n).$$

The determinants $\det D_{Mm}^c$ and $\det D_{Mm}^s$ are produced from $\det D_M$ by replacing the mth column by the constant term vector $\left[W_1^c, ..., W_M^c\right]^T$ or $\left[W_1^s, ..., W_M^s\right]^T$, respectively.

Taking [4] into account, the minimization of the function F exponent in likelihood function (2) will be replaced by the equivelent procedure which is maximization of a sum composed of cross products of the summands formed by squared residuals. Then, after substitution of signal amplitude estimates (4), we get

$$F_M = \sum_{m=1}^{M} \left\{ \frac{\det D_{Mm}^c}{\det D_M} \cdot W_m^c + \frac{\det D_{Mm}^s}{\det D_M} \cdot W_m^s \right\} = -\frac{\det \bar{D}_{Mm}}{\det D_M} \quad (5)$$

where

$$\det \bar{D}_{Mm} = \begin{vmatrix} 0 & \dot{W}_1^* \ \ldots \ \dot{W}_M^* \\ \dot{W}_1 & \\ \ldots & \det D_M \\ \dot{W}_M & \end{vmatrix}$$

and $\dot{W}_n^*$ means complex conjugate to $\dot{W}_n$.

Thus, the range measurement problem is reduced to maximization of function (5) on a set of feasible values of relative delay times of signals z_m, their number M, and the relative position of the signals in the time domain. The latter defines the structure of the determinant $\det D_M$ in which, without superposition of the mth and nth pulse, the elements outside the principal diagonal take zero values. Note that a priori we have no information about the overlapping of some or other echo signals, so for any assumed number M of sources we have to exhaust all the possible alternatives of $\det D_M$ beginning from purely diagonal when all entries off the principal diagonal are zeroes to the absolutely nonzero version. It is apparent that the case with all nondiagonal elements having zero values represents a classical problem of the Rayleigh resolution of targets in terms of their range. As for the ultimate choice of the $\det D_M$ pattern, the most acceptable version of the determinant will correspond to maximum of function F_M (5).

It is essential that all the $\det D_M$ components are calculable and in subsequent proceders may be read out from permanent memory

The approach considered may be easily extended to the case when the pulse duration is equal to that of the partial summation interval. However, the super-resolution of M sources in terms of range without picking up additional information about their parameters is possible only when the gating of the previous pulse terminatrion is also the gating of the next pulse beginning. With this scheme of echo signal superposition, the sums of the principal diagonal $\det D_M$ becomes zero for all $m - n \geq 2$.

Similar range-measuring procedures have been synthesized by the author of the paper for real video signals and complex radio pulses with arbitrary but a known pattern of the carrier parameter modulation.

Since the results obtained have no parallel it would be interesting to estimate the accuracy characteristics of the synthesized measurement procedures. We limit ourselves by evaluating the lower Cramer-Rao bound for the variance of a non-biased range estimation. Having used the matrix notation of the logarithmic equivalent of the likelihood functional, let us proceed to the following version of (2):

$$L = \mathrm{tr}\,\{\, [U - S(Z)\,A\,]^* \, [U - S(Z)\,A\,] \} \tag{6}$$

where "$*$" denotes complex-conjugate transposition, $U = [\dot{U}_1 \ \dot{U}_2 \ldots \dot{U}_T\ ]^T$ is the vector of complex voltages of the signals corresponding to T gatings; $A = [\dot{a}_1 \ \dot{a}_2 \ldots \dot{a}_M\ ]^T$ is the vector of complex amplitudes of the signals from M sources; and $S(Z)$ is a $T \times M$ matrix whose elements are determined by (1).

After differentiation L as a scalar function in conformity with (5) and subsequent matrix differentiation by Neudecker [6], we may determine the meanvalues of the second derivatives of scalar function (6):

$$E\left\{\frac{d^2 L}{d Z^2}\right\} = 2 \cdot [S'(Z)\,]^T \cdot (A\,A^* \otimes 1_T) \cdot S'(Z),$$

$$E\left\{\frac{d^2 L}{d A\, d Z}\right\} = 2 \cdot (A^* \otimes S^T(Z)) \cdot S'(Z),$$

$$E\left\{\frac{d^2 L}{d A^2}\right\} = 2 \cdot S^T(Z) \cdot S(Z),$$

$$E\left\{\frac{d^2 L}{d Z\, d A}\right\} = \left[E\left\{\frac{d^2 L}{d A\, d Z}\right\}\right]^* = 2\,[S'(Z)\,]^T \cdot (A \otimes S(Z)) \tag{7}$$

where $\otimes$ denotes direct (Kronecker) multiplication of matrices [6]; $S'(Z)$ is Neudecker's derivative of matrix $S(Z)$ with respect to vector Z; and 1_T is a $T \times T$ unit matrix.

With the account of (7), the lower Cramer-Rao bound of the variance of the unbiased estimate of range may be obtained by inverting the Fisher information matrix:

$$I = \frac{1}{\sigma^2} \cdot \begin{bmatrix} S^T(Z) \cdot S(Z) & (A^* \otimes S^T(Z)) \cdot S'(Z) \\ [S'(Z)]^T \cdot (A \otimes S(Z)) & [S'(Z)]^T \cdot (A A^* \otimes 1_T) \cdot S'(Z) \end{bmatrix}. \quad (8)$$

This case the variance of the estimate vector of the arrival times of signals Z may be determined from the vector $\mathrm{diag}(I^{-1})$ after eliminating the variances of the amplitude estimates.

To complete the mathematics, it remains to give concrete expressions for the blocks of matrix (8). To make the notation more compact we use all T gatings as summation limits:

$$S^T(Z) \cdot S(Z) = \begin{bmatrix} \sum_{t=1}^{T} S^2(z_1) & \sum_{t=1}^{T} S_t(z_1) S_t(z_2) & \dots & \sum_{t=1}^{T} S_t(z_1) S_t(z_M) \\ \vdots & \vdots & \dots & \vdots \\ \sum_{t=1}^{T} S_t(z_M) S_t(z_1) & \sum_{t=1}^{T} S_t(z_M) S_t(z_2) & \dots & \sum_{t=1}^{T} S_t^2(z_M) \end{bmatrix},$$

$$(A^* \otimes S^T(Z)) \cdot S'(Z) = \begin{bmatrix} a_1^* \cdot \begin{bmatrix} \sum_{t=1}^{T} S_t(z_1) S_t'(z_1) \\ \vdots \\ \sum_{t=1}^{T} S_t(z_M) S_t'(z_1) \end{bmatrix} & \dots & a_M^* \cdot \begin{bmatrix} \sum_{t=1}^{T} S_t(z_1) S_t'(z_M) \\ \vdots \\ \sum_{t=1}^{T} S_t(z_M) S_t'(z_M) \end{bmatrix} \end{bmatrix},$$

$$[S'(Z)]^T \cdot (A A^* \otimes 1_T) \cdot S'(Z) = \begin{bmatrix} a_1^2 \cdot \sum_{t=1}^{T} S_t'^2(z_1) & \dots & \dot{a}_1 a_M^* \cdot \sum_{t=1}^{T} S_t'(z_1) S_t'(z_M) \\ \vdots & \dots & \vdots \\ \dot{a}_M a_1^* \cdot \sum_{t=1}^{T} S_t'(z_M) S_t'(z_1) & \dots & a_M^2 \cdot \sum_{t=1}^{T} S_t'^2(z_M) \end{bmatrix}.$$

It follows from the above that in the case considered the accuracy of range measurement for each of M sources depends on the signal-to-noise ratio and – in a rather sophisticated manner - on signal overlapping degree, their biasing about the gating graticule, the envelope shape, and on the duration of the length of the ADC reading summation as well as on the number of gatings fitting within the pulse duration. It should be emphasized that the suggested approach is invariant, in terms of estimation accuracy, to a difference of initial phases of the interacting signals.

The obtained expression for Fisher's matrix is rather general and valid not only for multi-signal but also for single-signal range measurements. Particularly, in the case of a single radio pulse whose duration is equal to the gating time length, the lower Cramer-Rao bound of the bias estimate for the signal in the ADC readings about the beginning of the second signal gating takes on the form:

$$\sigma_{z_1}^2 \geq \frac{\sigma^2}{a_1^2} \cdot \left\{ \sum_{t=1}^{2} S_t'^2(z_1) - \frac{\left[\sum_{t=1}^{2} S_t(z_1) S_t'(z_1) \right]^2}{\sum_{t=1}^{2} S_t^2(z_1)} \right\}^{-1}$$

or, for a rectangular envelope and non-modulated carrier:

$$\sigma_{z_1}^2 \geq \frac{\sigma^2}{a_1^2} \cdot \frac{z_1^2 + (N - z_1)^2}{N^2} \quad (9)$$

where σ^2 is the variance of the summarized noise in the squared component of the gating response, and N is the gating duration in the ADC readings. According to (9), maximum accuracy may be achieved at the rectangular radio pulse shifted from the gating joint by a half of its duration. It is also obvious that, for the noise model assumed, that increasing the gating time length in the ADC readings will lead to deterioration of the range measurement accuracy due to growing the variance σ^2. As for detailed analysis of the accuracy properties for other reception conditions, it deserves a special consideration.

In conclusion it may be said that when the voltages are represented in the real form and the initial phase of radio pulses is unknown the above approach may entail considerable computational costs.

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