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LIMIT RESOLUTION OF RANGING PROCEDURES OF MAXIMUM LIKELIHOOD

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The paper considers the analytical method of estimating the limit resolution of the multisignal algorithms of maximum likelihood. The results are given of its use by the example of pulse ranging for signals with the envelope $\sin^2 x$.

In problems of spectral selection for checking the resolution of measuring procedures, artificial test sequences are used, which is determined [1] by the extreme complexity of the analytical description of characteristics of the majority of the estimation methods in multisignal situations. This leads to the impossibility of studying algorithms of super resolution by the analytical method without their experimental verification using sampling models of the signals. The objective of this article is consideration of analytical methods of detecting the potential resolution capacity of the maximum likelihood procedures using an example of pulse ranging as the least studied area of applying super resolution algorithms.

The distance b_{nm} between the neighboring sources will be understood as the resolution limit, whereby for the assigned signal-to-noise ratio the following condition is fulfilled:

$$b_{nm} \geq 3 \cdot \sigma_n + 3 \cdot \sigma_m, \tag{1}$$

where σ_n is the Cramer-Rao bound for MSE of the unbiased estimate of the nth source parameter which serves as resolution reference.

This indication makes it possible to secure ultimately low probability for the coincidence of estimates of the parameters of the signals constituting some $0.9 \cdot 10^{-3}$. From analysis (1) it follows that for the computation of the limiting resolution capacity of the method of maximum likelihood it is sufficient to determine the Cramer-Rao bound for the multisignal reception situation. Therefore, the departure point of the approach proposed is the derivation of the analytical expression for the limiting attainable variance of the errors when estimating the parameters M of interest of the sources.

To this end let us use the block representation of the Fisher information matrix **I** for the complex model of the signal and uncorrelated noises [2].

Let us confine ourselves to the case of in-phase interaction of pulses, which makes it possible to go in **I** to the real meanings of the values. E estimation error variances themselves, however, will be determined as a result of the inversion of matrix **I**, for instance, given a large number of signals using the method the augmented matrix [3].

With the assigned model of the signal and specific situation of their interaction it is necessary to compute the elements of the Fisher matrix and then carry out its inversion. Making the exhaustive search for possible versions of mutual location of the pulses and handling the signal-to-noise ratio we may find situations whereby condition (1) stops being fulfilled. This will actually become the limit of the resolution capacity for the assigned number of sources M.

In order to make initial conditions regular, let us confine ourselves to the equal signal-to-noise ratio for all signals and their uniform location in terms of range with the same length. In this case the resolution capacity in the two-signal situation is determined by the value $3\cdot(\sigma_1+\sigma_2)=6\cdot\sigma_{1(2)}$ since $\sigma_1=\sigma_2$. With $M \geq 3$ of the sources the result of the resolution is affected by estimation accuracy of the parameters of the internal group of signals, whose measuring conditions, given the large number of superimpositions and the sufficiently long pulse, are worst. Thus, given $M = 3$ the indication of the resolution is the MSE triple sum $\sigma_2+\sigma_{1(3)}$ since $\sigma_2>\sigma_3=\sigma_1$. For four equipotent sources $\sigma_2=\sigma_3>\sigma_1=\sigma_4$, therefore value $6\cdot\sigma_{2(3)}$ makes it possible to make a judgement about the quality of resolution. Finally, given the pulse, of, e.g., 100 samples and more, the five-signal situation is characterized with the relationship $\sigma_3\geq\sigma_2=\sigma_4\geq\sigma_1=\sigma_5$, which allows us, during the analysis of the resolution capacity, to be limited only by quality control $3\cdot(\sigma_3+\sigma_{2(4)})$.

To verify the correctness of how the procedures of inverting information matrices function it is advisable to use reference examples with the result predicted in advance as regards their various dimensions. Matrices A of the order n with the zero elements on the main diagonal and unities outside it and matrices reciprocal to them were used in the course of the study [4].

It is sufficient only to assign cross superimpositions of the time-neighboring pulses d_{nm} as initial data characterizing differences of the signals' time position, for the analysis of the limit resolution when measuring range directly using ADC samples. However, the depth of the cross superimposition of the rest is computed using the formulas: $d_{nk}=d_{nm}+d_{mk}-N$, where N is the length of the signals in ADC samples, $d_{nk}=N-b_{nk}$, b_{nk} is the distance between nth and kth sources. For example, for three signals, given known d_{12} and d_{23}, we will obtain: $d_{13}=d_{12}+d_{23}-N$.

Now let us discuss the result in the case of pulses with the envelope $\sin^2 x$. In contrast to [2] we take as a reference the measuring of the range without the additional clocking of ADC samples. In this case the value of the elements of the matrix S(z) [2] may be expressed in the form:

$$S_t(z_m)=\begin{cases}\sin^2(t-z_m)x & \text{at } z_m\leq t<z_m+N\\ 0 & \text{at } t<z_m \text{ and } t\geq z_m+N\end{cases},$$

where t is the ordinal number of an element of the matrix S(Z) in a column, t = 1, ..., z_M+N; z_m is the ordinal number of the first of the ADC samples, within the existence limits of the mth radio pulse, counted from the first signal $z_1=1$; N is the signal length in the ADC samples; m is the number of an element of the matrix S(Z) in a row; $x=\pi/N$.

Figure 1 represents the results of the investigations characterizing the ultimate resolution of two-sixth $\sin^2$-pulses having length on the basis of 100 ADC samples. The voltage signal-to-noise ratio is indicated along the horizontal axis; this voltage is assigned by several fold identical for all the signals; noise MSE in the form of the quadrature component were used as noise characteristics. The figures in the right-hand part of Fig. 1 commenting on the masks of the rectangles correspond to the number of permissible pulses. The distance between uniformly spaced sources, whereby condition (1) is satisfied, is plotted along the vertical axis in ADC samples. As should be expected, the resolution increases as the energy of the signals increases for any number. Given the voltage signal-to-noise ratio of 60 dB (1,024-fold) stable resolution is limited by the interval between neighboring pulses constituting 3–4 % of their length.

We noted almost complete coincidence of limit admissible intervals between the time-neighboring signals with $M > 4$ (taking into account rounding off). This is explained by the fact that given the above number of sources and their uniform distribution in terms of range, the MSE sum of time estimate for the arrival of the internal group signals in (1) varies a little as their number grows.

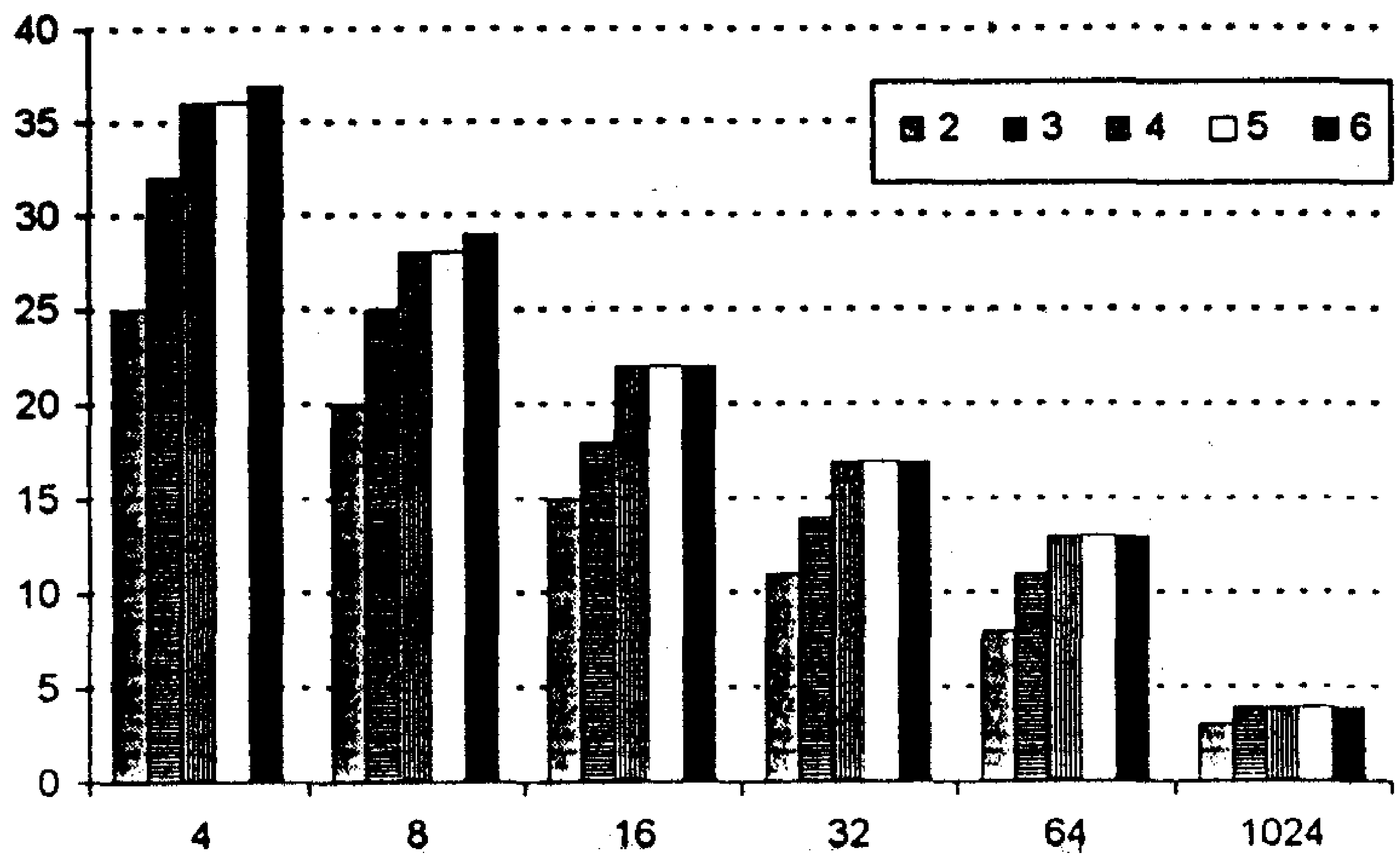

Fig. 1

Since one of the possible areas of applying range-measuring procedures of super resolution is the construction of range portraits of targets, the following is important.

When solving detection problems the analysis of the resolution characteristics should encompass not only MSE of range estimates, but also the errors in measuring the amplitudes of the signals. For instance, given the signal-to-noise ratio of 1,024 in the case of $M = 4$ the range limit resolution, as was indicated above, constitutes 4 % of the pulse length whereas MSE of the amplitudes of the signals reaches 15–29 %. Such accuracy is hardly acceptable for the qualitative construction of the target brightness portrait. At the same time, when doubling the interval between the signals the error of determining the amplitudes of internal pulses drops almost to 3.5 %. This should be taken into account when solving the problems of recalculating range estimation into the values of other parameters through the generalized signal amplitudes. Obviously, a large error of amplitude estimation of each pulse and hence generalized amplitudes will lead to apparent errors in measuring other coordinates.

One may also note that MSE in the estimation of time delays of some signals of the signal-to-noise ratio are independent of other signals. Given the nonuniform distribution of the sources in space, the resolution limits specified in Fig. 1 may also be reduced. The buildup of a portion of pulses within one time domain is "offset" by taking the rest of the signals at a large distance. For example, the resolution limit for three sources equal to 32 samples (signal-to-noise 4) may be reduced to 25 having "offset" a pair of signals with the interval of 25 by shifting the third pulse by 75 samples from the beginning of the middle signal.

The picture is maintained in terms of quality also with other lengths of the pulses. Thus, Fig. 2 shows the results corresponding to a 1,000-sample $\sin^2$ - signal. Compared with Fig. 1 one can see noted improvement of the resolution in percentage. Thus, given the signal-to-noise ratio of 1,024 in the two-signal situation the limit interval between the sources reduced up to 1.7 % of the pulse length. Such a result is in good compliance with the known one in spectral estimation by the 2 % resolution limit of the MUSIC procedure for the same energy [5].

It is interesting that in the three-signal situation to achieve the resolution limit inherent in the two-signal model, it is necessary to double the voltage signal-to-noise ratio. According to the data of Fig. 1 this regularity is not so expressly obvious. It is characteristic that such specific relation of two- and three-signal procedures of measurement is also maintained with other determinations of the resolution limit. Figure 3 for a 1000-sample signal with rectangles shows intervals satisfying the condition $b_{nm} \geq 2 \cdot \sigma_n + 2 \cdot \sigma_m$, with numerical notations remaining the same. Hence one may come to a fundamental conclusion on the expediency of opposing the two-target algorithm with a three-target one having the same resolution limit only after reducing the radar-group target distance $\sqrt{2}$ -fold. Similar adaptation may substantially reduce hardware costs necessary for the implementation of multisignal processing.

An important outcome of the investigations conducted is confirmation of the possibility of resolution according to condition (1) of 5–6 sources uniformly located in the pulse volume ADC 100 samples long, if the distance from them to the radar does not exceed $0.5 \cdot D_{\max}$. Here $D_{\max}$ is the range from which using a standard feed in the radar receiver

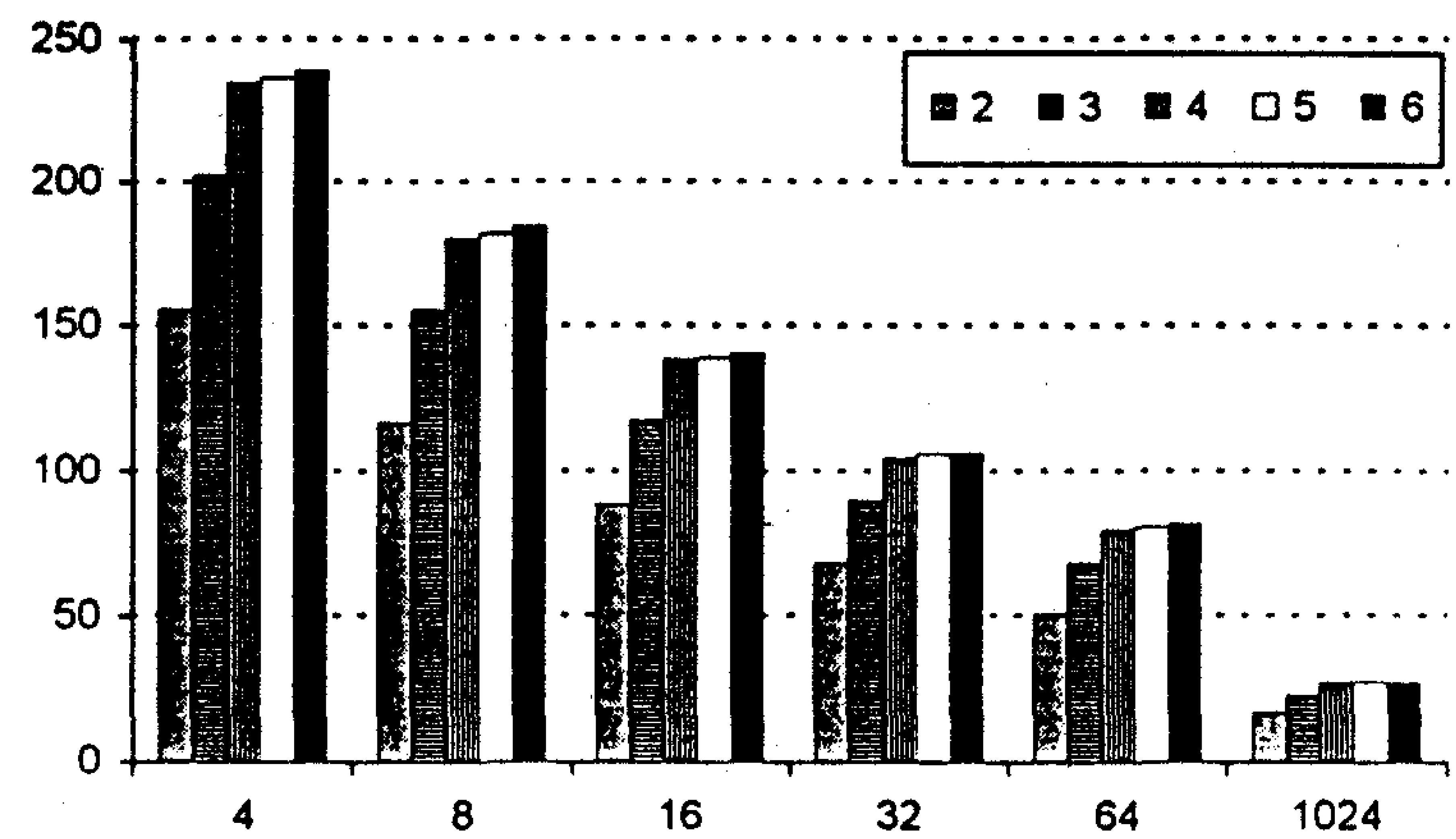

Fig. 2

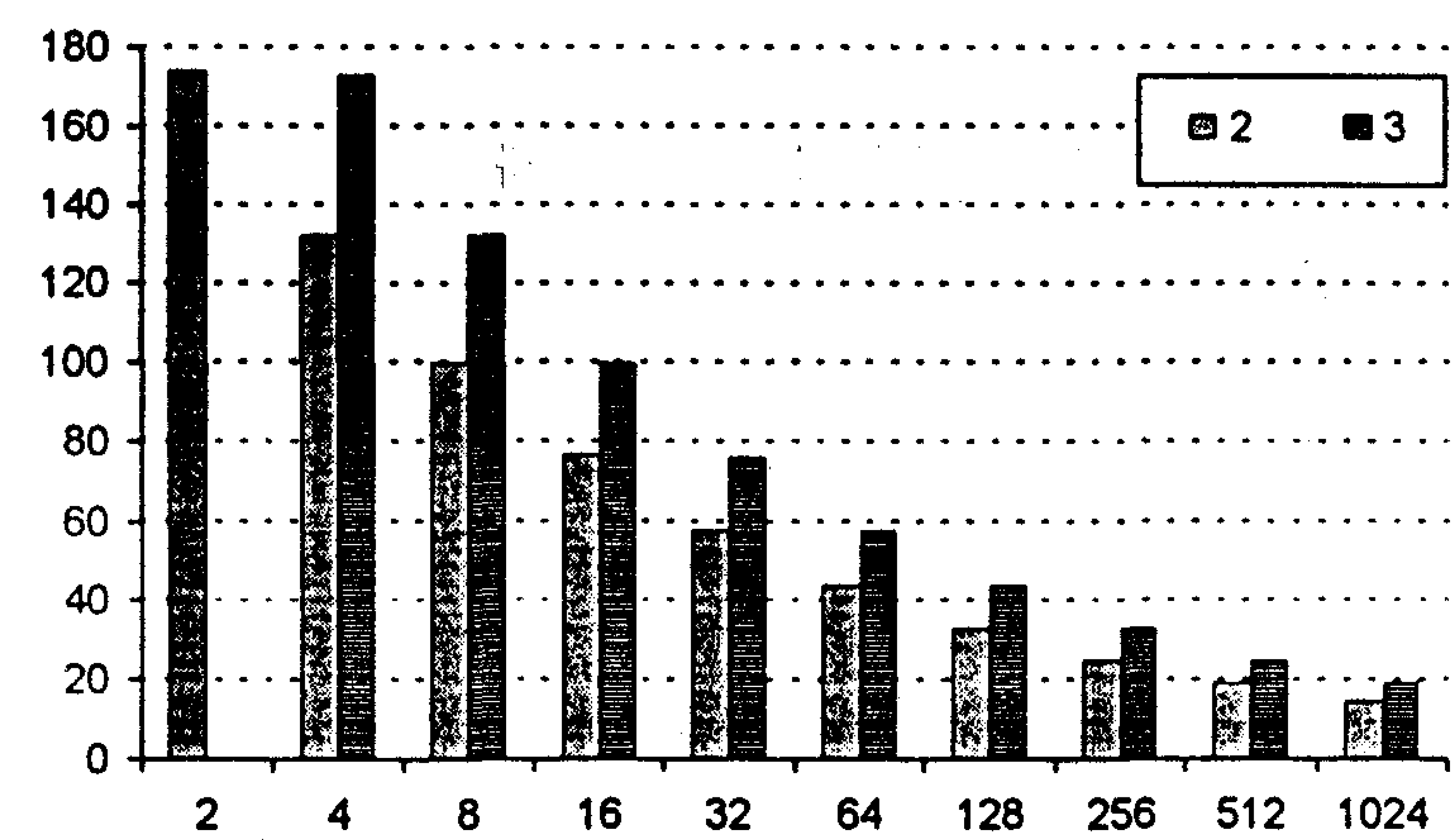

Fig. 3

we may ensure the voltage signal-to-noise ratio of 12 dB. Given a denser "packing" of echo signals, for instance, with the uniform interval of 13 % of their length, the resolution area of 5–6 pulses for a 100-sample signal is reduced to $0.25 \cdot D_{max}$.

Thus, the method considered makes it possible to reasonably distribute, in terms of distance, boundaries of setting the problems of resolutions of the M sources, varying the range value M. In addition, with its aid, it becomes possible to restrict the limits of iteration exhaustive search for time estimates of signal arrival depending on the amplitude of the signal mixture.

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