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END PRODUCTS IN MATRICES IN RADAR APPLICATIONS

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This paper introduces the concepts of the end product and transposed end product of matrices and their modifications on whose basis we obtained the records of the response of multicoordinate radars with digital antenna arrays.

Definition 1. We will refer to the matrix $A \square B$ of dimension $p \times gs$ – being determined by equality $A \square B = [a_{ij} \cdot B_i]$ – as the end product $p \times g$ – the matrices $A = [a_{ij}]$ and $p \times s$ – the matrices B represented as the block-matrix of rows B_i, $(B = [B_i], i = 1, ..., p)$.

As an example we can indicate the analytical model of the response of the multicoordinate radar based on the linear digital antenna array (DAA). Let us agree that such contains R reception channels with directivity characteristics $Q_r(x)$, where x is the direction at the radiation source, with T Doppler filters being synthesized at the output of each reception channel having amplitude-frequency characteristics $F_t(\omega)$, where ω is frequency. Upon the action of the sources of signals having complex amplitudes $\dot{a}_m$ at the input of such system M of signal sources, angular coordinates x_m and frequencies ω_m, noise-free voltage at the output of the tth frequency filter of the rth reception channel will represent the sum

$$\dot{U}_{tr} = \sum_{m=1}^{M} \dot{a}_m \cdot F_t(\omega_m) \cdot Q_r(x_m). \tag{1}$$

If to form a matrix of dimension $M \times R$ of the characteristics of the directivity of reception channels $[Q_j(x_i)]; j = 1, 2, ..., R; i = 1, 2, ..., M$, the $M \times T$-matrix of the AFC of the Doppler filters $[F_j(\omega_i)]; j = 1, 2, ..., T; i = 1, 2, ..., M$ and the vector of the complex amplitude signals

$$A = [\dot{a}_1 \; \dot{a}_2 \; ... \; \dot{a}_M]^{\mathrm{T}},$$

then expression (1) can be written in matrix form by means of the end product of matrices:

$$U = Q^{\mathrm{T}} (A \square F), \tag{2}$$

with elements (1).

In a similar way the response of the three-coordinate radar is formalized with the flat equidistant DAA:

$$U = Q^{\mathrm{T}} (A \square F \square V) \tag{3}$$

where V – $M \times R$-matrix of the directivity characteristics $V_r(y_m)$ R of the reception channels in the additional coordinate plane, U is the block matrix of the form $U = [U_1, ..., U_n, ..., U_R]$, with elements $\dot{U}_{trn} = \sum_{m=1}^{M} \dot{a}_m \cdot F_t(\omega_m) \cdot Q_r(x_m) \cdot V_n(y_m)$.

Having supplemented the direction-finding Doppler selection by the measurement of ranging [1] based on (3) we will write down the model of the four-coordinate array

$$U = Q^{\mathrm{T}} (A \square F \square V \square S) \tag{4}$$

where $S - M \times D$-matrix of responses D of the ranging strobes obtained as a result of the additional gating of ADC samples by accumulation with reset [1]. In this case in contrast to (3) the block structure of matrix (4) manifests periodicity determined by the presence of the fourth subscript.

It should be noted that for two-coordinate case (2) there is an alternative model within the framework of the traditional matrix algebra related to the artificial technique of "tension" of the vector of the amplitude of signals A on the main diagonal of the identity $M \times M$-matrix.

Corresponding analog (2) has the form [2]

$$U = Q^{\mathrm{T}} \cdot \mathrm{diag}\, [a_i] \cdot F, \; i = 1, 2, ..., M.$$

However, during transition to three- and four-coordinate models the known set of matrix operations becomes ineffective.

The end product introduced here holds an intermediate niche between the Hadamard product [3] and direct (Kronecker, tensor) product of the matrices [4]. Its name reflects the fact that the right-hand matrix before multiplication by the elements of the left one looks as if it is split from the end into rows.

Following the principle of symmetry, definition I can be given in the form $A \square B = [A_i \cdot b_{ij}]$. Since in both cases we obtain concepts with identical properties, both definitions could be equally useful in applications as well. However, the circuit $[a_{ij} B_i]$ is preferably closer to the direct product [4].

The associativity of the matrix end product and its separation property with respect to addition are checked very easily:

$$(A \square B) \square C = A \square (B \square C),$$
$$(A + B) \square C = A \square C + B \square C,$$
$$A \square (B + C) = A \square B + A \square C,$$
$$(A + B) \square (C + D) = A \square C + B \square C + A \square D + B \square D.$$

These expressions imply that the number of rows of the first and the second cofactors coincide.

As the normal matrix product, the end product is noncommutative ($A \square B \neq B \square A$) although for the vector commutativity is acceptable $a \square b = b \square a$.

In many applications a property coupling the end and direct products of the square matrices may be useful

$$A \otimes (B \square C) = (A \otimes B) \square C.$$

It is characteristic that for the Hadamard product such associativity is impossible:

$$A \circ (B \square C) \neq (A \circ B) \square C.$$

Observance of necessary dimension of co-factors is a defining factor also for the conversion of the end product result. The operation $(A \square B)^{-1}$ makes sense if for the $p \times g$-matrix A and $p \times s$-matrix B the identity $p = s \times g$ is valid. Otherwise only conversion according to Penrose is possible.

Finally, the law of the order conversion is interesting [3]. If for the normal matrix product it is formulated in the form of $(A \cdot B)^{\mathrm{T}} = B^{\mathrm{T}} \cdot A^{\mathrm{T}}$, then in the case of the end product it is required to introduce a new concept: the transposed end product (TEP).

Definition 2. We will refer to the $g\,s \times p$-matrix $A \blacksquare B$ being determined by the equality

$$A \blacksquare B = [a_{ij} \cdot B_j].$$

as the transposed end product of the $g \times p$-matrix $A = [a_{ij}]$ and $s \times p$-block matrix of the columns $B = [B_j], j = 1, ..., p$.

Transposition of the result of the end product will be written as:

$$(A \square B)^{\mathrm{T}} = A^{\mathrm{T}} \blacksquare B^{\mathrm{T}}.$$

Using TEP we can propose an alternative version for solving the problem of analytical modeling of the DAA response. Instead of relationships (2)–(4) for the same matrices Q, F, V, S and the vector A we will obtain

$$\tilde{U}_{(2)} = (Q^{T} \blacksquare F^{T}) \cdot A, \tag{5}$$

$$\tilde{U}_{(3)} = (Q^{T} \blacksquare F^{T} \blacksquare V^{T}) \cdot A, \tag{6}$$

$$\tilde{U}_{(4)} = (Q^{T} \blacksquare F^{T} \blacksquare V^{T} \blacksquare S^{T}) \cdot A. \tag{7}$$

Convenience of TEP for signal processing and the analysis of the accuracy of multicoordinate systems is in the possibility of using the results obtained within the framework of the traditional set of actions over the matrices as applied to one- or two-coordinate radars. For instance, using the calculations of [1] and having denoted $P = Q^{T} \blacksquare F^{T} \blacksquare V^{T} \blacksquare S^{T}$ it is not difficult to write down the Fisher information matrix for the characteristic of the maximum attainable accuracy of model (7)

$$I = \frac{1}{\sigma^2} \cdot \begin{bmatrix} P^{T} \cdot P & \vdots & (A^{*} \otimes P^{T}) \cdot P'_x \\ \cdots & \cdot & \cdots \\ [P'_x]^{T} (A \otimes P) & \vdots & [P'_x]^{T} \cdot (A A^{*} \otimes 1_{TRRD}) P_x \end{bmatrix}$$

where P is the Noidecker derivative of the matrix P along the vector X composed from unknown parameters of the signals [4], 1_{TRRD} is the identity matrix of dimension $T \times R \times R \times D$, $\otimes$ is the product sign.

Going to the extremely complex problem on the basis of newly introduced types of products of matrices we can formalize the response of the multiposition system of W conformly four-coordinate DAA containing G sections each.

Differences between the sections and radar positions in the characteristics of directivity, AFC of the filters and responses of the strobes of the range will be expressed having replaced the matrices F^{T}, Q^{T}, V^{T} and S^{T} with block structures having $G \times W$ blocks corresponding to different sections and multiposition differences of the parameters will be arranged along the vertical. In this case, for instance, in place of the matrix of characteristics Q^{T} in (5) – (7) we will obtain the block-matrix

$$Q^{T}_{GW} = \begin{bmatrix} \tilde{Q}_{11} \vdots \tilde{Q}_{21} \vdots \ldots \vdots \tilde{Q}_{G_1} \vdots \ldots \vdots \tilde{Q}_{GW} \end{bmatrix}^{T},$$

$$\tilde{Q}_{nm} = \begin{bmatrix} Q_{1nm}(x_1) & Q_{2nm}(x_1) & \ldots & Q_{Rnm}(x_1) \\ Q_{1nm}(x_2) & Q_{2nm}(x_2) & \ldots & Q_{Rnm}(x_2) \\ \vdots & \vdots & & \vdots \\ Q_{1nm}(x_M) & Q_{2nm}(x_M) & \ldots & Q_{Rnm}(x_M) \end{bmatrix}$$

The block-matrices F^{T}_{GW}, V^{T}_{GW}, S^{T}_{GW} will attain a similar break-down into blocks coordinates with Q^{T}_{GM}. Further calculations require the introduction of the concepts of the block end product (BEP) and BEP transposition.

Definition 3. The matrix $A \circledcirc B$ being determined by the equality

$$A \circledcirc B = [A_{ij} \Box B_{ij}]. \tag{8}$$

will be referred to as the block end product $bp \times cs$-matrix $A = [A_{ij}]$ and $bp \times cg$-matrix $B = [B_{ij}]$ ($i = 1, \ldots, b; j = 1, \ldots, c$) with the coordinated break-down into blocks of dimension $p \times s$ and $p \times g$, respectively.

The BEP sign $\circledcirc$ symbolizes the circumstance that the blocks of the matrices having the same name are taken to fulfill end multiplication ($\Box$) using the principle of the Hadamard product. Introduction of a separate modification of the end product in place of imposing restrictions on its properties preserves the possibility of end multiplication of the block-matrices with uncoordinated break-down into blocks.

Definition 4. The matrix $A \circledbullet B$ being determined by the equality

$$A \circledbullet B = [A_{ij} \blacksquare B_{ij}]$$

will be accordingly referred to as the transposed block end product (TBEP) $cs \times bp$-matrix $A = [A_{ij}]$ and $cg \times bp$-matrix $B = [B_{ij}]$ ($j = 1, ..., c$; $i = 1, ..., b$) with matched breakdown into blocks of dimension $s \times p$ and $g \times p$.

Let us formalize the response of the multiposition radar system

$$\tilde{U}_{(4\,G\,W)} = (Q^{T}_{GW} \circledbullet F^{T}_{GW} \circledbullet V^{T}_{GW} \circledbullet S^{T}_{GW}) \cdot A. \tag{9}$$

In the considered case TBEP made it possible to implement the formation of the DAA four-coordinate response for each of G sections W of the radar positions.

The possibility of combination within one record of both the end and transposed end product with their block modifications should be noted. Such a technique will make it possible to conveniently pack the multiplication result into a multiblock system in contrast to the vector representation of the voltage arrays obtained in (5)–(9).

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