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MEASUREMENT OF THE REPETITION PERIOD OF PULSES OVERLAPPING OVER TIME

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The paper proposed digital methods of measuring the period of pulse train in high-speed packages by the efficiency of narrowband processing systems.

Under conditions of fast growth in the speed of digital signal processing systems it is problematic to maintain that pace in the development of diagnostic means of controlling their parameters. Therefore, it is justified to adapt narrowband equipment to the solution of new problems.

Let us consider a version of solving the given problem by using more sophisticated methods of estimating sought-for parameters using as an example the measuring of the period of the train pulse signals in high-speed synchronous packets. The use of narrowband input circuits in this case leads to the merging of the output response of the measuring unit into a solid signal from whose appearance it is not possible to make a judgement about parameters of periodicity of the input pulse train. It makes sense to consider key aspects of a new approach after declaration of the main assumptions within whose framework it is appropriate to make subsequent formalization and statement of the problem of the synthesis of estimation methods.

All the following calculations are oriented to processing several narrowband pulse signals or their packets. In this case it is assumed that such pulses have identical known shape of the envelope and within the interval of the reception of a signal packet the period of their repetition remains invariable. We will also assume that the narrowband receiving channel introduces negligibly small nonlinear distortions and its output response may be considered as linear superposition of output signals.

In the case of a packet of M video pulses of equal amplitude the given assumptions make it possible to record noiseless voltages of the signal mixture by the efficiency of the narrowband channel (after their analog-to-digital conversion) in the following form:

$$U_s = a \sum_{m=0}^{M=1} K_m (z + s + m \cdot d), \tag{1}$$

where a is amplitude of signals, $K_m (z + s + m \cdot d)$ is the discrete function of the envelope of the mth video pulse in the sth number sample of pulse sampling whose beginning is shifted by z samples with respect to the beginning of the first of the signals of the packet, d is the interval of pulse repetition in fractions of digitization period.

Within the context of the problem solved one should consider the value d as the sought-for unknown in (1). Subsequent calculations for the synthesis of estimation methods of such will be conducted oriented to the presence of all M signals in each of the sth sample and $\sin^2$ of the envelope for which (1) will be rewritten in the form:

$$U_s = a \sum_{m=0}^{M-1} \sin^2 \left(\frac{\pi (z + s + m \cdot d)}{N} \right), \tag{2}$$

where N is the duration of a pulse by the base of the envelope in ADC samples.

Taking into account [1], having denoted $y_0 = \pi / N$ for even and odd M from (2) we will obtain

$$U_s = a\left(\frac{M}{2} - \frac{\cos[y_0(2z + 2s + (M-1)d)] \cdot \sin(My_0 d)}{2\sin(y_0 d)}\right). \quad (3)$$

To reduce the problem of determining the unknown d to the solution of an algebraic system of equations it remains to collect such a number of samples U_s whereby the sought-for system of equalities lends itself to normalization. In the simplest case (taking into account the unknown amplitude of signals and shift of the measuring sampling z) for the packet of real video pulses considered it is sufficient to use three ADC samples taken, for example, in a row rather than within an interval of T samples:

$$U_k = a\left(\frac{M}{2} - \frac{\cos[y_0(2z + 2(k-1)T + (M-1)d)] \cdot \sin(My_0 d)}{2\sin(y_0 d)}\right), \; k = 1, 2, 3.$$

Among the multitude of approaches to the solution of such a system of equations we will use exclusion of the unknown amplitude a by pair-by-pair normalization that form the system of equations. Having denoted $y = 2z + (M-1)d$ after conversion we will obtain:

$$\frac{U_2 - U_1}{U_3 - U_1} = \frac{U_2\cos(y_0 y) - U_1\cos[y_0(y + 2T)]}{U_3\cos(y_0 y) - U_1\cos[y_0(y + 4T)]}, \quad (4)$$

$$\frac{\sin(My_0 d)}{M\sin(y_0 d)} = \frac{U_2 - U_1}{U_2\cos(y_0 y) - U_1\cos[y_0(y + 2T)]}.$$

The first of these equations may be solved with respect to the unknown y after reducing it to the form

$$\tan(y_0 y) = \frac{U_2 - U_3\beta + U_1\beta\cos(4Ty_0) - U_1\cos[2Ty_0)}{U_1\beta\sin(4Ty_0) - U_1\sin[2Ty_0]},$$

where $\beta = (U_2 - U_1)/(U_3 - U_1)$, from the second equality by the table function $(\sin(My_0 d))/(M\sin(y_0 d))$ the sought-for estimate d is determined.

Results of checking the measuring procedure discussed in MathCad 7.0 confirmed its efficiency and the possibility of determining the values d 100-1000 times smaller than the ADC digitization period. This substantially expands the boundaries of using the given method, in particular, with respect to speed selection of fast-moving sources in radar technology, where the value d may be interpreted as the shift of an echo signal over the repetition period of probing pulses.

In situations when it is impossible to guarantee the presence of M signals in ADC samples selected for measurement, the procedure of estimating their repetition period should be supplemented by verification of the hypothesis relative to actual presence in them of one number of pulses or another. In the deterministic interpretation of the signal mixture it is enough to find among the multimode of admissible values M and d such a pair of them whereby the square of difference between the left-hand and right-hand parts of the second equation in (4) will be minimal.

If there is a necessity to take into account measuring noises for the synthesis of the estimation procedure the method of the least squares (MLS) may be recommended. Applicable to the S-reading measuring sample the corresponding functional of discrepancies according to (2) and (3) will be written in the form

$$F = \sum_{s=1}^{S}\left\{U_s - a\left(\frac{M}{2} - \frac{\cos[y_0(2z + 2(s-1)T + (M-1)d)] \cdot \sin(My_0 d)}{2\sin(y_0 d)}\right)\right\}^2 = \min. \quad (5)$$

In this case for verifying hypotheses on the number of the signals M the information equivalent may be used

$$F_M = \sum_{s=1}^{S}\left\{U_s \cdot a\left(M - \frac{\cos[y_0(2z + 2(s-1)T + (M-1)d)] \cdot \sin(My_0 d)}{\sin(y_0 d)}\right)\right\} = \max,$$

which taking into account the amplitude optimal estimate will be rewritten in the form

$$F_M = \frac{2\left[\sum_{s=1}^{S} U_s \left(M - \frac{\cos\left[y_0 (2z + 2(s-1)T + (M-1)d)\right] \cdot \sin(M y_0 d)}{\sin(y_0 d)}\right)\right]^2}{\sum_{s=1}^{S} \left(M - \frac{\cos\left[y_0 (2z + 2(s-1)T + (M-1)d)\right] \cdot \sin(M y_0 d)}{\sin(y_0 d)}\right)^2} = \max$$

or for the random number M in each sample

$$\widetilde{F}_M = \frac{2\left[\sum_{s=1}^{S} U_s \left(M_s - \frac{\cos\left[y_0 (2z + 2(s-1)T + (M_s-1)d)\right] \cdot \sin(M_s y_0 d)}{\sin(y_0 d)}\right)\right]^2}{\sum_{s=1}^{S} \left(M_s - \frac{\cos\left[y_0 (2z + 2(s-1)T + (M_s-1)d)\right] \cdot \sin(M_s y_0 d)}{\sin(y_0 d)}\right)^2} = \max.$$

As a result the process of testing the hypotheses is reduced to searching for the values $M(M_s)$ and d maximizing the expression F_M ($\widetilde{F}_M$).

For deriving an optimal estimate of the train period d it is convenient instead of (5) to handle the sum of discrepancies obtained by normalization of the sth sample to the first one in the reference sampling, i.e.

$$F_1 = \sum_{s=1}^{S} \left\{1 - \frac{U_1}{U_{1+s}} - \frac{\sin(M y_0 d)}{M \sin(y_0 d)} \left[\cos(y_0 y) - \frac{U_1}{U_{1+s}} \cos\left[y_0 (y + 2sT)\right]\right]\right\}^2 = \min, \quad (6)$$

and also by the function of discrepancies formed according to a similar principle based on the first equation of system (4), viz.

$$F_2 = \sum_{s=1}^{S} \left\{(U_1 \cdot \beta_s \sin\left[2(s+1)Ty_0\right] - U_1 \sin 2Ty_0) \cdot \mathrm{tg}(y_0 y) - C_s\right\}^2 = \min, \quad (7)$$

where

$$C_s = U_1 \beta_s \cos 2(s+1)Ty_0 - U_1 \cos 2Ty_0 + U_2 - U_{s+2}\beta_s,$$
$$\beta_s = (U_2 - U_1)/(U_{s+2} - U_1).$$

From (6) it follows

$$\frac{\sin(M y_0 d)}{M \sin(y_0 d)} = \frac{\sum_{s=1}^{S} \left(1 - \frac{U_1}{U_{1+s}}\right)\left[\cos(y_0 y) - \frac{U_1}{U_{1+s}} \cos\left[y_0 (y + 2sT)\right]\right]}{\sum_{s=1}^{S} \left[\cos(y_0 y) - \frac{U_1}{U_{1+s}} \cos\left[y_0 (y + 2sT)\right]\right]^2} \quad (8)$$

and minimizing (7) we obtain

$$\tan(y_0 y) = \frac{\sum_{s=1}^{S} \left(\beta_s \cos p_s - \cos 2Ty_0 + \frac{U_2 - U_{s+2}\beta_s}{U_1}\right)(\beta_s \sin p_s - \sin 2Ty_0)}{\sum_{s=1}^{S} \left\{\beta_s \sin p_s - \sin 2Ty_0\right\}^2}, \quad (9)$$

where $p_s = 2(s+1)Ty_0$.

Table 1

σ_z	$\sigma_d \cdot 10^{-3}$	T	S
0.438	0.827	24	8
0.310	0.591	12	16
0.219	0.402	6	32
0.155	0.298	3	64
0.090	0.172	1	192

The estimate d from (8) taking into account (9) with large signal-to-noise ratios slightly differs in terms of accuracy from that corresponding to the functional of the Cramer–Rao boundary for the variance of a measurement error. It may be obtained as a lower element of the main diagonal of the matrix inverse to the Fischer matrix

$$I = \frac{1}{\sigma_n^2} \cdot \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}, \tag{10}$$

where

$$I_{11} = \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} \sin^2 A \right]^2, \quad I_{22} = B^2 \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} \sin 2A \right]^2, \quad I_{33} = B^2 \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} m \sin 2A \right]^2,$$

$$I_{12} = B \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} \sin^2 A \sum_{m=0}^{M-1} \sin 2A \right], \quad I_{13} = B \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} \sin^2 A \sum_{m=0}^{M-1} m \sin 2A \right],$$

$$I_{23} = B^2 \sum_{s=0}^{S-1} \left[\sum_{m=0}^{M-1} \sin 2A \sum_{m=0}^{M-1} m \sin 2A \right], \quad A = \pi\,(z + s\,T + m\,d)\,/\,N, \;\; B = a\,\pi\,/N.$$

Investigation of factors affecting potential accuracy of measuring the period d conducted using calculations by relationship (10) showed that its growth occurs when the following parameters increase: the signal-to-noise ratio; number of pulses at the input packet M; number S used for measuring samples of the signal mixture; interval T between them with the fixed dimension S; pulse train period d; speed of the signal increase and decrease controlled in this case by the duration N of the $\sin^2$ pulse; shift of the beginning of the measuring sampling z with respect to the instant of the emergence of the first of the signals of the packet (z should not exceed 40% of the $\sin^2$ pulse duration since the discontinuous nature of function (9) may lead to an opposite effect).

The conclusion that in order to increase measurement accuracy d it is preferable to increase the number of samples of the measuring sampling S, bringing them together in time, rather than to increase the interval T between samples to the detriment of their number, was not very obvious. An effect achievable in this case is illustrated in Table 1, which represents root-mean-square errors of measuring the values z (σ_z) and d (σ_d) with a single signal-to-noise ratio, different values T, S, and their fixed product $T \times S = 192$. Other initial data were set by the following: pulse duration $N = 256$, number of pulses in a packet $M = 1024$, the train period of input signals $d = 0.04$, the shift of the first sample of the sampling with respect to the beginning of a packet $z = 5$.

In a more complex formulation the measuring problem considered implies the processing of signals having, in the general case, an amplitude varying from pulse to pulse. Coming back again to ADC output codes in such interpretation relationship (2) should be rewritten in the form

$$U_s = a \sum_{m=0}^{M-1} \beta_m \sin^2\left(\frac{\pi\,(z+s+m\cdot d)}{N}\right), \tag{11}$$

where β_m is the coefficient taking into account the deviation of the amplitude of the mth signal from the reference value.

In order to determine d "against the background" of indefiniteness of amplitudes of signals and the shift of the first reading of the measuring sampling z, in the general case, one needs at least $M+2$ samples (11) within the interval of covering the signals. The given limitations narrow the range of admissible pulse repetition periods without providing for as great as possible increase of the duration of the signal packet with increasing efficiency of the final estimation result. For the weakening of the given limitation one may recommend another class of measuring procedures based on the use of the additional gating of ADC samples [2]. Such processing at the expense of accumulation of samples obtained at different times makes it possible in a number of cases to ensure superposition of signal even in the absence of their actual overlap.

In conclusion we will note that in a similar way the problem of changing the train period may be solved also for signals with other forms of the envelope including that of a nonanalytical form.

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