29

CONTENTS

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THE METHOD OF NONORTHOGONAL FREQUENCY-DISCRETE MODULATION OF SIGNALS FOR NARROW-BAND COMMUNICATION CHANNELS

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The suggested computational procedures make it possible to perform frequency-division multiplexing of narrow-band communication channels based on the nonorthogonal discrete modulation of signals.

The method of orthogonal frequency-discrete modulation of signals (OFDM), which has found wide utility in communication systems, is intended for relatively wide-band information highways. In the case of narrow-band communication channels, OFDM loses its appeal.

The purpose of this paper is to consider an alternative to the OFDM method, which consists in nonorthogonal frequency modulation of signals and permits using a denser arrangement of carriers in the spectral domain.

As in traditional OFDM, assume that at the transmitting end the shaping of the signal, in conformity with the frequency-discrete and quadrature-phase modulation (QPM) principles, is carried out with the aid of a signal processor and a digital-to-analog converter. However, as distinct from the OFDM method, the frequency shift will not be associated with the maxima of amplitude-frequency responses of the filters, which ought to be synthesized at the reception end by means of the fast Fourier transform.

The procedure of signal demodulation in the receiver of messages will be derived under the assumption that the instant of arrival of the signal burst is known exactly. With the use of the maximum likelihood method and provided that the noise is noncorrelated and has the Gaussian distribution, we set up the informational equivalent of the likelihood function for the real-valued representation of the sum of M harmonic signals

$$L=\sum_{s=1}^{S}\left\{U_s-\sum_{m=1}^{M}a_m\cos\left(\omega_m\Delta t(s-1)+\varphi_m\right)\right\}^2 \tag{1}$$

or, if denoting $a_m^c=a_m\cos\varphi_m, a_m^s=a_m\sin\varphi_m, p_{ms}=\omega_m\Delta t(s-1)$,

$$L=\sum_{s=1}^{S}\left\{U_s-\sum_{m=1}^{M}\left(a_m^c\cos p_{ms}-a_m^s\sin p_{ms}\right)\right\}^2=\min, \tag{2}$$

where U_s is the signal mixture voltage in the sth time sample, a_m is the amplitude of the mth harmonic signal, S is the total number of time samples to be processed ($S\geq 2M$), s is the ordinal number of ADC reading within the signal sample, ω_m is the circular frequency of the mth signal, and φ_m is its initial phase.

Here we assume that the amplitude of the signals remains unchanged during S samplings. As for the amplitude samples, they can be obtained by solving the likelihood equations

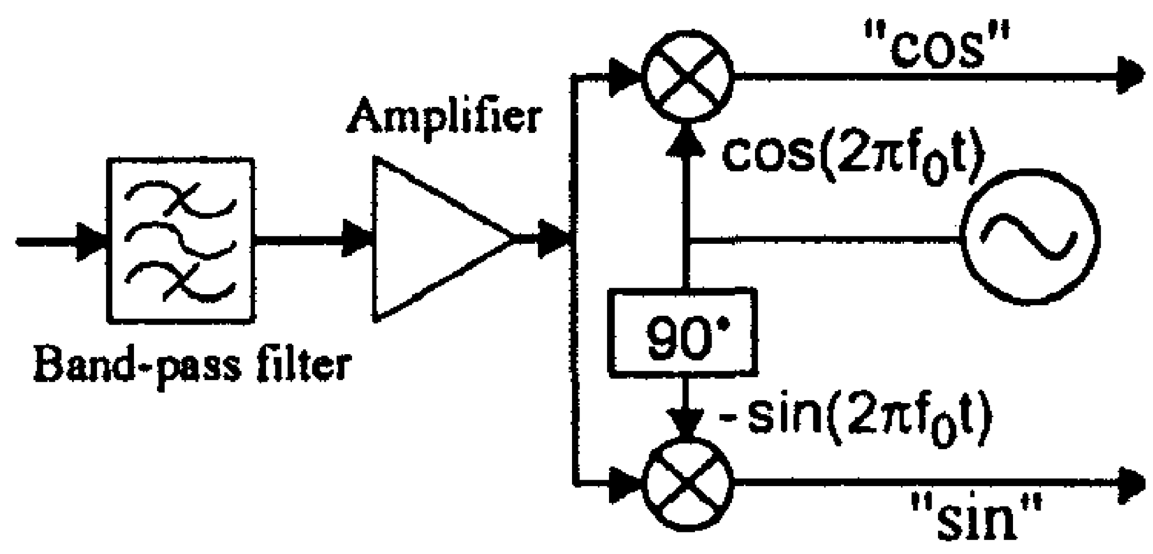

Fig. 1

$$\partial L / \partial a_m^c = 0; \ \partial L / \partial a_m^s = 0. \tag{3}$$

Based on Cramer's rule, the solution to linear algebraic equation systems can be represented as

$$a_m^c = D_m^c / D; \ a_m^s = D_m^s / D, \tag{4}$$

where

$$D = \begin{vmatrix} B_{11} & C_{12} & \dots & C_{1M} \\ C_{21} & B_{22} & \dots & C_{2M} \\ \vdots & \vdots & \dots & \vdots \\ C_{M1} & C_{M2} & \dots & B_{MM} \end{vmatrix}, B_{nn} = \begin{bmatrix} \sum_{s=1}^{S} \cos^2 p_{ns} & -0.5\sum_{s=1}^{S} \sin 2p_{ns} \\ -0.5\sum_{s=1}^{S} \sin 2p_{ns} & \sum_{s=1}^{S} \sin^2 p_{ns} \end{bmatrix},$$

$$C_{nm} = \begin{bmatrix} \sum_{s=1}^{S} \cos p_{ns} \cos p_{ms} & -\sum_{s=1}^{S} \cos p_{ns} \sin p_{ms} \\ -\sum_{s=1}^{S} \cos p_{ms} \sin p_{ns} & \sum_{s=1}^{S} \sin p_{ns} \sin p_{ms} \end{bmatrix}, C_{nm} = C_{mn}^{\mathrm{T}}.$$

The determinants D_m^c, D_m^s are produced from the determinant D by replacing the respective mth even (for D_m^c) or odd (for D_m^s) column by the vector of free terms

$$\begin{bmatrix} W_1^c & W_1^s & W_2^c & W_2^s & \dots & W_M^c & W_M^s \end{bmatrix}^T$$

where $W_m^c = -\sum_{s=1}^{S} U_s \cos p_{ms}$, $W_m^s = \sum_{s=1}^{S} U_s \sin p_{ms}$.

Thus, the problem of determination of quadrature components of the amplitudes of the sum of M harmonic signals, represented in the real form, was reduced to determination of estimates (2) by substitution of known values p_{ms}. To make the computation simpler and to minimize the number of spurious reception channels, it is desirable to use the complex-valued representation of signals. The hardware implementation of the analog "dequadratization" reduces to application of a reference generator and a signal multiplication device — as shown in Fig. 1 in [1].

After analog-to-digital conversion of every generated quadrature, the procedure of measurement of the amplitude components can be obtained by minimization of the functional

$$L_M = \sum_{s=1}^{S} \left[\left\{ U_s^c - \sum_{m=1}^{M} a_m \cos(\omega_m \Delta t(s-1) + \varphi_m) \right\}^2 + \left\{ U_s^s - \sum_{m=1}^{M} a_m \sin(\omega_m \Delta t(s-1) + \varphi_m) \right\}^2 \right] \tag{5}$$

where U_s^c, U_s^s are quadrature components of the signal mixture voltage at the ADC output in the sth signal sample, or, using the notation given in (2),

$$L_M = \sum_{s=1}^{S}\left[\left\{U_s^c - \sum_{m=1}^{M}\left(a_m^c \cos p_{ms} - a_m^s \sin p_{ms}\right)\right\}^2 + \left\{U_s^s - \sum_{m=1}^{M}\left(a_m^c \sin p_{ms} + a_m^s \cos p_{ms}\right)\right\}^2\right].$$

In order to form the estimates of quadrature components of the amplitudes, we must substitute the following values into (4):

$$C_{nm} = \begin{bmatrix} \sum_{s=0}^{S-1}\cos(\omega_m - \omega_n)s\Delta t & -\sum_{s=0}^{S-1}\sin(\omega_m - \omega_n)s\Delta t \\ \sum_{s=0}^{S-1}\sin(\omega_m - \omega_n)s\Delta t & \sum_{s=0}^{S-1}\cos(\omega_m - \omega_n)s\Delta t \end{bmatrix}, \quad B_{nn} = \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix},$$

and, instead of the vector of free terms, respectively,

$$W_m^c = \sum_{s=1}^{S}\left\{U_s^c \cos \omega_m s\Delta t + U_s^s \sin \omega_m s\Delta t\right\}$$

and

$$W_m^s = \sum_{s=1}^{S}\left\{U_s^s \cos \omega_m s\Delta t - U_s^c \sin \omega_m s\Delta t\right\}.$$

The use of the lower Cramer-Rao bound permits us to evaluate the potential accuracy of our method in estimating the quadrature components of amplitudes. To make the mathematics simpler, we shall apply to matrix notation [2] and represent the vector of ADC real samples, used in (1), in the "noiseless" form

$$U = F \cdot A \tag{6}$$

where $U = [U_1 \quad U_2 \quad \dots \quad U_S]^{\mathrm{T}}$ is the vector of real voltages of the signal mixture,

$$A = \left[a_1^c \quad a_1^s \quad a_2^c \quad a_2^s \quad \dots \quad a_M^c \quad a_M^s\right]^{\mathrm{T}}$$

is the vector of quadrature components of signals' amplitudes, and

$$F = \begin{bmatrix} \cos p_{11} & -\sin p_{11} & \cos p_{21} & -\sin p_{21} & \dots & \cos p_{M1} & -\sin p_{M1} \\ \cos p_{12} & -\sin p_{12} & \cos p_{22} & -\sin p_{22} & \dots & \cos p_{M2} & -\sin p_{M2} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ \cos p_{1S} & -\sin p_{1S} & \cos p_{2S} & -\sin p_{2S} & \dots & \cos p_{MS} & -\sin p_{MS} \end{bmatrix}.$$

Here the optimal estimate of the amplitude quadrature components, used for decoding the messages, can be obtained in a more concise (than in (4)) matrix form:

$$A = \left\{F^{\mathrm{T}}F\right\}^{-1} F^{\mathrm{T}}U. \tag{7}$$

Using for our case the expression for Fisher's matrix, derived in [2] based on the Neudecker derivative, we may write that

$$I = (1/\sigma_{\mathrm{n}}^2) \cdot F^{\mathrm{T}}F \tag{8}$$

where the elements of the quadratic form $F^{\mathrm{T}}F$ are identical to the entries of the determinant D in (4).

In the event of a single-frequency signal,

$$F^{\mathrm{T}}F=\begin{bmatrix} \sum\limits_{s=1}^{S}\cos^2 p_{ns} & -0.5\cdot\sum\limits_{s=1}^{S}\sin 2p_{ns} \\ -0.5\cdot\sum\limits_{s=1}^{S}\sin 2p_{ns} & \sum\limits_{s=1}^{S}\sin^2 p_{ns} \end{bmatrix}.$$

The required estimates of the variance of amplitude components σ_a^2 can be obtained as diagonal elements of the inverse Fisher matrix after inverting the matrix (8).

In the case of complex-valued representation of the vector of digital samples of signal mixture used in (5), the components of matrix expression (6) take the form

$$U=\begin{bmatrix} \dot{U}_1 \\ \dot{U}_2 \\ \vdots \\ \dot{U}_S \end{bmatrix},\quad F=\begin{bmatrix} \exp(jp_{11}) & \exp(jp_{21}) & \dots & \exp(jp_{M1}) \\ \exp(jp_{12}) & \exp(jp_{22}) & \dots & \exp(jp_{M2}) \\ \vdots & \vdots & \vdots & \vdots \\ \exp(jp_{1S}) & \exp(jp_{2S}) & \dots & \exp(jp_{MS}) \end{bmatrix},\quad A=\begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \vdots \\ \dot{a}_M \end{bmatrix}. \tag{9}$$

As a result, the Fisher information matrix

$$I=F^{*}F/\sigma_{\mathrm{n}}^2 \tag{10}$$

where

$$F^{*}F=\begin{bmatrix} S & \sum\limits_{s=1}^{S}\exp[j(p_{2s}-p_{1s})] & \dots & \sum\limits_{s=1}^{S}\exp[j(p_{Ms}-p_{1s})] \\ \sum\limits_{s=1}^{S}\exp[j(p_{1s}-p_{2s})] & S & \dots & \sum\limits_{s=1}^{S}\exp[j(p_{Ms}-p_{2s})] \\ \vdots & \vdots & \dots & \vdots \\ \sum\limits_{s=1}^{S}\exp[j(p_{1s}-p_{Ms})] & \sum\limits_{s=1}^{S}\exp[j(p_{2s}-p_{Ms})] & \dots & S \end{bmatrix},$$

and "$*$" means complex conjugation.

For a solitary harmonic signal, $M=1$ and $F^{*}F=S$.

The estimates of quadrature components of the amplitudes can be defined, by analogy with (7), in the form

$$A^{c}=\mathrm{Re}\left(\left\{F^{*}F\right\}^{-1}\cdot F^{*}\cdot U\right)\; A^{s}=\mathrm{Im}\left(\left\{F^{*}F\right\}^{-1}\cdot F^{*}\cdot U\right)$$

where $A^{c}=\begin{bmatrix}a_1^c & a_2^c & \dots & a_M^c\end{bmatrix}^{\mathrm{T}}$, $A^{s}=\begin{bmatrix}a_1^s & a_2^s & \dots & a_M^s\end{bmatrix}^{\mathrm{T}}$.

To retain uniformity in representation of the vector of amplitudes introduced in (7), the complex expression of the model of signal mixture (9) can be modified:

$$\dot{U}=\begin{bmatrix}U_1^c & U_1^s & U_2^c & U_2^s & \dots & U_M^c & U_M^s\end{bmatrix}^{\mathrm{T}};$$

$$F=\begin{bmatrix} \cos p_{11} & -\sin p_{11} & \cos p_{21} & -\sin p_{21} & \dots & \cos p_{M1} & -\sin p_{M1} \\ \sin p_{11} & \cos p_{11} & \sin p_{21} & \cos p_{21} & \dots & \sin p_{M1} & \cos p_{M1} \\ \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ \cos p_{1S} & -\sin p_{1S} & \cos p_{2S} & -\sin p_{2S} & \dots & \cos p_{MS} & -\sin p_{MS} \\ \sin p_{1S} & \cos p_{1S} & \sin p_{2S} & \cos p_{2S} & \dots & \sin p_{MS} & \cos p_{MS} \end{bmatrix};$$

$$A=\begin{bmatrix}a_1^c & a_1^s & a_2^c & a_2^s & \dots & a_M^c & a_M^s\end{bmatrix}^{\mathrm{T}}.$$

In this case the vector A estimates and the Fisher information matrix can be obtained by the same formulas (7) and (8) as for real representation of the signal mixture. The lower Cramer-Rao bound calculated by expressions (8) and (10) makes it possible to evaluate potentialities of the suggested methods of demodulation at frequency multiplexing of signals.

To take a further look at the σ_a^2 behavior, we have performed a computational experiment in the Mathcad medium. Comparison of potential accuracy of the method represented by (7) with the lower Cramer-Rao bound calculated for the OFDM methods and for its nonorthogonal modification [3] shows that at the same number of samplings, identical carriers and equal noise values, their accuracy is the same. However, as distinct from the above decisions, the new method of demodulation can operate with an arbitrary number of samples, not necessarily equal to the integral power of 2. In addition, the new method permits reducing the computational expenditures due to preliminary calculation of the projecting operator $\{F^{\mathrm{T}}F\}^{-1} \cdot F^{\mathrm{T}}$ for known frequencies of the signals, and abandonment of the fast Fourier transform procedure.

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