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Methods for Estimating the ADC Jitter in Noncoherent Systems

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Abstract—The paper presents the method of estimating the jitter dispersion developed for the case when the clock pulse generator of ADC and the generator of input test signal are not time synchronized. The generalization of the method was carried out for the following cases: input signal represents a sum of harmonic components; ADC system has several channels with equal values of the jitter and additive noise dispersions. The results of numerical simulation are also presented.

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The methods considered in literature for jitter estimation in systems with analog-to-digital converters (ADC), as a rule, imply (in explicit or implicit form) that the device generating a test signal and the clock pulse generator of ADC are time synchronized and so ensure the repetition of events (for example, [1, 2]). This ensures the formation of statistical characteristics of samples (for example, dispersion and mathematical expectation) that are obtained under the equal conditions. However, in practice, the high-quality clocked generators (synthesizers) are not always available due to their high cost. At the same time, the price of high-quality free-running generators for fixed frequencies is much lower. In addition, the methods using the clocked generators (synthesizers) as sources of a test signal cannot be always applied when the tested system represents a complete device and the in-built generator cannot be used as the master generator of the test system (for example due to its low quality).

The purpose of this study is to develop the methods for estimating the jitter dispersion in systems with ADC under an assumption that the signal source is not synchronized with the clock generator of ADC and also to analyze the usability conditions of the specified methods.

Let the following harmonic signal be present at the input of jitter ADC:

$$u(t) = A\sin(2\pi F t + \varphi), \tag{1}$$

where A, φ, and F are the amplitude, initial phase, and frequency of the input signal, respectively.

Let us write down the results of signal sampling at the ADC output in the form of expansion into the Taylor series in the neighborhoods of the sampling instants with preservation of terms of the first order of smallness assuming the jitter is small:

$$u_i \approx A\sin(2\pi f i + \varphi) + 2\pi F \tau_i A\cos(2\pi f i + \varphi) + \eta_i, \tag{2}$$

where i is the sample number, f is the ratio of the input signal frequency to the sampling frequency, τ_i is the jitter during the generation of the ith sample, η_i is the value of additive noise during the generation of the ith sample. Next we shall assume that the jitter and additive noise samples are independent and have the average values equal to zero and dispersions σ_τ^2 and σ_η^2, respectively. We shall also assume that during the entire duration of observations dispersions σ_τ^2 and σ_η^2 remain constant.

Let M samplings be observed, and each of them contains N samples. In this case it is assumed that the amplitude and frequency of the input signal do not vary from sampling to sampling, while the initial phase of samplings is a random quantity due to the absence of synchronization of the input signal generator and clock pulse generator and, therefore, has the uniform distribution in the range from $-\pi$ to π. Using the representation of signal voltage samples in the form of (2), we shall write down the average value of power of the mth sampling ($m=\overline{0, M-1}$ in the form:

$$P_m = E\left\{\sum_{i=0}^{N-1} u_{i,m}^2\right\} = P_{S,m} + \sigma_\tau^2 P_{D,m} + N\sigma_\eta^2, \tag{3}$$

where $u_{i,m}$ is the ith sample of the mth sampling, $P_{S,m}$ is the power of harmonic component of the mth sampling, $\sigma_\tau^2 P_{D,m}$ is the noise power caused by jitter, $P_{D,m}$ is the square of the signal first derivative taken in terms of the jitter samples at the instants of sampling:

$$P_{S,m} = A^2 \sum_{i=0}^{N-1} \sin^2(2\pi f i + \varphi_m), \tag{4}$$

$$P_{D,m} = 4\pi^2 F^2 A^2 \sum_{i=0}^{N-1} \cos^2(2\pi f i + \varphi_m), \tag{5}$$

where φ_m is the initial phase of harmonic signal in the mth sampling.

Using the known trigonometric identities

$$\cos^2\alpha = 2^{-1}(1+\cos 2\alpha),$$

$$\sin^2\alpha = 2^{-1}(1-\cos 2\alpha) \tag{6}$$

and formula from paper [3, item 1.341, subitem 1]

$$\sum_{i=0}^{n-1} \cos(x+ky) = \cos\left(x + \frac{n-1}{2}y\right)\sin\left(\frac{ny}{2}\right)\sin^{-1}\left(\frac{y}{2}\right), \tag{7}$$

expressions for $P_{S,m}$ and $P_{D,m}$ can be rewritten in the form

$$P_{S,m} = A^2 2^{-1}(N - \cos(2\pi f(N-1) + 2\varphi_m)\sin(2\pi f N)\sin^{-1}(2\pi f)), \tag{8}$$

$$P_{D,m} = 4\pi^2 F^2 A^2 2^{-1}(N + \cos(2\pi f(N-1) + 2\varphi_m)\sin(2\pi f N)\sin^{-1}(2\pi f)). \tag{9}$$

Expression (9) reflects the fact that in the general case the fraction of the noise power of observed sampling generated by jitter depends on the initial phase of signal in the sampling.

In order to determine the jitter dispersion in terms of the results of observation by using the least squares method on the basis of expressions (3)–(5), we shall write down the following goal function:

$$G_1 = \sum_{m=0}^{M-1} (\hat{P}_m - \hat{P}_{S,m} - \sigma_\tau^2 \hat{P}_{D,m} - N\sigma_\eta^2)^2 = \min, \tag{10}$$

where $\hat{P}_m$ is the estimate of the total power of the mth sampling, $\hat{P}_{S,m}$ is the estimate of the power of the sine component, $\hat{P}_{D,m}$ is the estimate of the square of the signal first derivative. Estimate $\hat{P}_m$ is calculated in terms of samples of the mth sampling. Estimates $\hat{P}_{S,m}$ and $\hat{P}_{D,m}$ are calculated on the basis of estimates of the signal amplitude and initial phase in the mth sampling by using expressions (4) and (5) or more effectively in computational sense by using expressions (8) and (9).

Differentiating function G_1 with respect to σ_τ^2 and σ_η^2 we obtain the system of two equations:

$$\begin{cases} B_1 - \sigma_\tau^2 B_2 - N\sigma_\eta^2 B_3 = 0, \\ B_4 - \sigma_\tau^2 B_3 - NM\sigma_\eta^2 = 0, \end{cases} \tag{11}$$

where

$$B_1 = \sum_{m=0}^{M-1} \hat{P}_{D,m}(\hat{P}_m - \hat{P}_{S,m}), \quad B_2 = \sum_{m=0}^{M-1} \hat{P}_{D,m}^2, \tag{12}$$

$$B_3 = \sum_{m=0}^{M-1} \hat{P}_{D,m}, \quad B_4 = \sum_{m=0}^{M-1}(\hat{P}_m - \hat{P}_{S,m}). \tag{13}$$

The solution of system (11) with respect to σ_τ^2 and σ_η^2 yields the expressions for estimates of the jitter dispersion and additive noise

$$\hat{\sigma}_\tau^2 = (MB_1 - B_3B_4)(MB_2 - B_3^2)^{-1},$$

$$\hat{\sigma}_\eta^2 = (B_2B_4 - B_1B_3)N^{-1}(MB_2 - B_3^2)^{-1}. \tag{14}$$

The usability condition of the method developed is specified by the following expression:

$$P_{D,m} \neq P_{D,n}, \varphi_m \neq \varphi_n, \quad n, m = \overline{0, M-1}. \tag{15}$$

In case condition (15) is not fulfilled, i.e.

$$\forall \varphi_m, \varphi_n : P_{D,m} = P_{D,n} = P_D, \; n, m = \overline{0, M-1},$$

it can be shown that system (11) degenerates into the system of identical equations of the form

$$\sum_{m=0}^{M-1}(\hat{P}_m - \hat{P}_{S,m}) - \sigma_\tau^2 MP_D - NM\sigma_\eta^2 = 0, \tag{16}$$

and the estimates of dispersions cannot be obtained.

Taking into account expression (9) the usability condition of method (15) can be written in the refined form:

$$\sin(2\pi fN)\sin^{-1}(2\pi f) \neq 0. \tag{17}$$

Now we shall extend the developed estimation method to the case when the input signal represents a sum of sine waves of the known frequency:

$$u(t) = \sum_{k=1}^{K} A_k \sin(2\pi F_k t + \varphi_k), \tag{18}$$

where K is the number of harmonic components of different frequencies in the input signal, A_k, F_k, and φ_k are the amplitude, frequency, and initial phase of the kth harmonic component of the input signal, respectively.

Assuming the jitter is small, we shall write down the result of sampling at the ADC output in the form of the Taylor series expansion in the neighborhoods of sampling instants preserving the terms of the first order of smallness:

$$u_{i,m} \approx \sum_{k=1}^{K} A_k \sin(2\pi f_k i + \varphi_{k,m}) + 2\pi\tau_i \sum_{k=1}^{K} A_k F_k \cos(2\pi f_k i + \varphi_{k,m}) + \eta_i, \tag{19}$$

where f_k is the ratio of the kth component frequency of the input signal to the sampling frequency.

We shall assume that the average value of power of the mth sampling, similar to the previous case, is specified by expression (3). In this case quantities $P_{S,m}$ and $P_{D,m}$ having the invariable physical meaning are described by the following expressions:

$$P_{S,m} = \sum_{i=0}^{N-1} \left(\sum_{k=1}^{K} A_k \sin(2\pi f_k i + \varphi_{k,m}) \right)^2, \tag{20}$$

$$P_{D,m} = 4\pi^2 \sum_{i=0}^{N-1} \left(\sum_{k=1}^{K} A_k F_k \cos(2\pi f_k i + \varphi_{k,m}) \right)^2, \tag{21}$$

where $\varphi_{k,m}$ is the initial phase of the kth component of harmonic signal in the mth sampling.

The estimate of jitter dispersion still can be obtained by the least squares method. In this case the goal function can be written in the form of expression (10). It can be shown that expressions for dispersion estimates σ_τ^2 and σ_η^2 have the form of (14). Expressions for quantities B_1, B_2, B_3 and B_4 entering the composition of estimates are described by expressions (12) and (13) (with due regard for the fact that estimates $\hat{P}_{S,m}$ and $\hat{P}_{D,m}$ are calculated in accordance with expressions (20) and (21)).

In order to derive the usability condition of the method, we shall take into account the following identity:

$$\left(\sum_{k=1}^{K} a_k \right)^2 = \sum_{k=1}^{K} a_k^2 + 2 \sum_{p=1}^{K-1} \sum_{k=p+1}^{K} a_k a_k. \tag{22}$$

Then, in the case of multifrequency input signal, using identities (22), (6), and (7) we can write expression (21) for $P_{D,m}$ in the form:

$$P_{D,m} = 2\pi^2 \sum_{k=1}^{K} A_k^2 F_k^2 \left(N + \cos\big(2\pi f_k (N-1) + 2\varphi_{k,m}\big) \frac{\sin(2\pi f_k N)}{\sin(2\pi f_k)} \right)$$

$$+ 4\pi^2 \sum_{p=1}^{K-1} \sum_{q=p+1}^{K-2} A_p F_p A_q F_q \cos\big(\pi(f_p - f_q)(N-1) + \varphi_{p,m} - \varphi_{q,m}\big) \frac{\sin\big(\pi(f_p - f_q)N\big)}{\sin\big(\pi(f_p - f_q)\big)}$$

$$+ 4\pi^2 \sum_{p=1}^{K-1} \sum_{q=p+1}^{K-2} A_p F_p A_q F_q \cos\big(\pi(f_p + f_q)(N-1) + \varphi_{p,m} + \varphi_{q,m}\big) \frac{\sin\big(\pi(f_p + f_q)N\big)}{\sin\big(\pi(f_p + f_q)\big)}. \tag{23}$$

From expression (23) the following usability condition of the method is obtained: at least one of the following expressions should be valid:

$$\sin(2\pi f_k N)\sin^{-1}(2\pi f_k) \neq 0 \quad (k=\overline{1,K}), \tag{24}$$

$$\sin\left(\pi(f_p - f_q)N\right)\sin^{-1}\left(\pi(f_p - f_q)\right) \neq 0$$

$$(p=\overline{1,K-1}, q=\overline{p+1,K}), \tag{25}$$

$$\sin\left(\pi(f_p + f_q)N\right)\sin^{-1}\left(\pi(f_p + f_q)\right) \neq 0$$

$$(p=\overline{1,K-1}, q=\overline{p+1,K}). \tag{26}$$

If a system of analog-to-digital conversion contains several channels of conversion, which (for example due to their circuit and design solutions) can be considered as identical from the viewpoint of values of the jitter dispersion and additive noise, goal function (10) can be modified for the simultaneous use of data of all channels in obtaining the estimates of dispersions. In this case the goal function, irrespective of using the single-frequency or multifrequency method, has the form:

$$G_2 = \sum_{q=0}^{Q-1}\sum_{m=0}^{M-1}(\hat{P}_{m,q} - \hat{P}_{S,m,q} - \sigma_\tau^2 \hat{P}_{D,m,,q} - N\sigma_\eta^2)^2 = \min, \tag{27}$$

where $\hat{P}_{m,q}$ is the estimate of the total power of the mth sampling in the qth channel, $\hat{P}_{S,m,q}$ is the estimate of the power of sine component (or sum of components) in the qth channel, $\hat{P}_{D,m}$ is the estimate of the square of the first derivative of signal in the qth channel. Estimate $\hat{P}_{m,q}$ is calculated from samples of the sampling. Estimates $\hat{P}_{S,m,q}$ and $\hat{P}_{D,m,q}$ are calculated on the basis of estimates of amplitudes and initial phases of the signal components.

Differentiating function G_2 with respect to σ_τ^2 and σ_η^2 we shall form a system of two equations:

$$\begin{cases} R_1 - \sigma_\tau^2 R_2 - N\sigma_\eta^2 R_3 = 0, \\ R_4 - \sigma_\tau^2 R_3 - NQM\sigma_\eta^2 = 0, \end{cases} \tag{28}$$

where

$$R_1 = \sum_{q=0}^{Q-1}\sum_{m=0}^{M-1}\hat{P}_{D,m,q}(\hat{P}_{m,q} - \hat{P}_{S,m,q}), \qquad R_2 = \sum_{q=0}^{Q-1}\sum_{m=0}^{M-1}\hat{P}_{D,m,q}^2, \tag{29}$$

$$R_3 = \sum_{q=0}^{Q-1}\sum_{m=0}^{M-1}\hat{P}_{D,m,q}, \qquad R_4 = \sum_{q=0}^{Q-1}\sum_{m=0}^{M-1}(\hat{P}_{m,q} - \hat{P}_{S,m,q}). \tag{30}$$

Solving the obtained system (28) with respect to σ_τ^2 and σ_η^2 we shall obtain expressions for estimates of the jitter and additive noise dispersions:

$$\hat{\sigma}_\tau^2 = (MQR_1 - R_3R_4)(MQR_2 - R_3^2)^{-1},$$

$$\hat{\sigma}_\eta^2 = (R_2R_4 - R_1R_3)N^{-1}(MQR_2 - R_3^2)^{-1}. \tag{31}$$

The estimation of amplitudes and initial phases of harmonic components in the samplings (in the case of the multifrequency method) may involve the use of a technique proposed in paper [4]. In this case the estimation of jitter dispersion can be combined with measurement of the amplitude-frequency characteristics.

In addition, the least-squares method can be used for the estimation of amplitudes and initial phases of harmonic components. In this case the quadrature components of amplitudes of harmonic components of the *m*th sampling are estimated at the first stage. The expressions in the matrix form are as follows [5]:

$$\mathbf{Q} = \{\mathbf{F}^{\mathrm{T}}\mathbf{F}\}^{-1}\mathbf{F}^{\mathrm{T}}U, \tag{32}$$

where $\mathbf{Q} = [q_1^s q_1^c q_2^s q_2^c \ldots q_K^s q_K^c]^{\mathrm{T}}$ is the vector of quadrature components of signal amplitudes of the *m*th sampling, $\mathbf{U} = [u_0 u_1 \ldots u_{N-1}]^{\mathrm{T}}$ is the vector of voltage samples of the *m*th sampling,

$$\mathbf{F} = \begin{bmatrix} \cos p_{1,0} & \sin p_{1,0} & \cos p_{2,0} & \sin p_{2,0} & \cdots & \cos p_{K,0} & \sin p_{K,0} \\ \cos p_{1,1} & \sin p_{1,1} & \cos p_{2,0} & \sin p_{2,1} & \cdots & \cos p_{K,1} & \sin p_{K,1} \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ \cos p_{1,N-1} & \sin p_{1,N-1} & \cos p_{2,N-1} & \sin p_{2,N-1} & \cdots & \cos p_{K,N-1} & \sin p_{K,N-1} \end{bmatrix},$$

$$p_{k,i} = 2\pi f_k i \quad (k = \overline{1,K}, i = \overline{0, N-1}).$$

Next the amplitudes and initial phases of harmonic components are estimated by using the quadrature components:

$$\hat{A}_k = \sqrt{(q_k^c)^2 + (q_k^s)^2}, \quad \hat{\varphi}_k = \arctan\frac{q_k^s}{q_k^c}. \tag{33}$$

In case the frequencies of the input signal components cannot be assumed known, it is necessary to perform a prior measurement of frequencies (for example by using the methods presented in paper [6]). However, it should be taken into account that due to the increasing number of estimated parameters, the estimation accuracy of jitter dispersion will decline.

The operability of the developed methods was tested by numerical simulation.

Table 1 presents the conditions and results of numerical experiments for the single-frequency method. The generation of samplings of samples involved the use of the following conditions:

1) the amplitude of harmonic signal was equal to 1000 quanta of ADC;

2) root-mean-square deviation (RMSD) of jitter σ_τ was specified in fractions of the input signal period;

3) additive noise RMSD σ_η was specified in ADC quanta;

4) sample distributions of the additive noise and jitter were assumed to obey the normal law.

100 samplings were used to obtain one pair of estimates $\hat{\sigma}_\tau^2$ and $\hat{\sigma}_\eta^2$. The length of each sampling was equal to 100 samples. Table 1 presents the following results: sampling average $E\{\hat{\sigma}_\tau^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\tau^2\}}$ for $\hat{\sigma}_\tau^2$, sampling average $E\{\hat{\sigma}_\eta^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\eta^2\}}$ for $\hat{\sigma}_\eta^2$. The sampling average and RMSD were estimated on the basis of a set of 1000 experiments.

From the results of Table 1 one can make the following conclusions:

1) At $\sigma_\tau > 10^{-3}$ we observe the appearance of the bias of estimator $\hat{\sigma}_\eta^2$ and the rise of RMSD $\sqrt{D\{\hat{\sigma}_\eta^2\}}$ making the joint estimation of the jitter and additive noise dispersions (positions Nos. 3–5) impossible.

2) The estimation accuracy depends on the absolute value of quantity $\sin(2\pi f N)\sin^{-1}(2\pi f)$. Its rise by the factor of about 9 results in the RMSD reduction by the factor of almost 8 (Nos. 2, 3, 7–9). This makes it

Table 1

No.	f	σ_τ	σ_η	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\eta^2\}$	$\sqrt{D\{\hat{\sigma}_\eta^2\}}$	$\frac{\sin(2\pi fN)}{\sin(2\pi f)}$
1	1.991	10^{-4}	1.1	1.07×10^{-8}	1.36×10^{-8}	1.17	0.27	–10.4
2	1.991	10^{-3}	1.1	9.89×10^{-7}	2.39×10^{-7}	1.00	4.70	–10.4
3	1.991	10^{-2}	1.1	9.42×10^{-5}	2.40×10^{-5}	80.95	4.71×10^{2}	–10.4
4	1.991	5×10^{-2}	1.1	7.18×10^{-4}	5.02×10^{-4}	3.32×10^{4}	9.01×10^{3}	–10.4
5	1.999	10^{-2}	1.1	9.65×10^{-5}	3.11×10^{-6}	33.14	37.52	93.55
6	1.999	10^{-2}	100	9.12×10^{-5}	1.31×10^{-5}	9.92×10^{3}	2.79×10^{2}	93.55
7	1.999	10^{-3}	1.1	9.81×10^{-7}	2.87×10^{-8}	1.16	0.34	93.55
8	1.501	10^{-3}	1.1	9.81×10^{-7}	2.93×10^{-8}	1.18	0.34	93.55
9	1.501	10^{-2}	1.1	9.61×10^{-5}	3.10×10^{-6}	38.58	36.97	93.55
10	3.496	8.74×10^{-4}	1.5	7.49×10^{-7}	8.62×10^{-8}	2.20	1.65	–23.39

Table 2

No.	f_1	f_2	σ_τ	$\left\lvert\frac{\sin(2\pi f_1 N)}{\sin(2\pi f_1)}\right\rvert$	$\left\lvert\frac{\sin(2\pi f_2 N)}{\sin(2\pi f_2)}\right\rvert$	$\left\lvert\frac{\sin(2\pi(f_1+f_2)N)}{\sin(2\pi(f_1+f_2))}\right\rvert$	$\left\lvert\frac{\sin(2\pi(f_1-f_2)N)}{\sin(2\pi(f_1-f_2))}\right\rvert$
1	1.991	1.977	10^{-3}	10.4	6.4	4.76	6.69
2	1.991	1.977	10^{-2}	10.4	6.4	4.76	6.69
3	1.991	1.999	10^{-3}	10.4	93.55	0	18.9
4	1.991	1.999	10^{-2}	10.4	93.55	0	18.9
5	1.991	1.51	10^{-3}	10.4	0	93.55	4.94
6	1.991	1.51	10^{-2}	10.4	0	93.55	4.94
7	1.991	1.501	10^{-3}	10.4	93.55	100	75.68
8	1.991	1.501	10^{-2}	10.4	93.55	100	75.68

possible to choose the number of samples in a sampling for the maximization of the specified quantity during the performance of measurements.

3) The method preserves its operability at $\sigma_\eta = 100$ that amounts to 0.1 amplitude of the input signal (position No. 6), in this case estimate dispersion $\sqrt{D\{\hat{\sigma}_\tau^2\}}$ is less than in experiment No. 3 when $\sigma_\eta = 1.1$. This is determined by a larger value of quantity $\sin(2\pi fN)\sin^{-1}(2\pi f)$.

Table 2 presents the conditions of numerical experiments for the multifrequency method. The following conditions were used during the generation of samplings of samples:

1) the signal consisted of two harmonic components;

2) amplitudes of both harmonic components were equal to 1000 quanta of ADC;

Table 3

No.	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\eta^2\}$	$\sqrt{D\{\hat{\sigma}_\eta^2\}}$
1	9.82×10^{-7}	1.31×10^{-7}	0.28	5.00
2	9.76×10^{-5}	1.31×10^{-5}	–55. 1	501.5
3	9.69×10^{-7}	5.81×10^{-8}	0.82	1.90
4	8.86×10^{-5}	6.36×10^{-6}	279.2	213.3
5	9.58×10^{-7}	4.59×10^{-7}	1.19	14.2
6	8.87×10^{-5}	4.40×10^{-5}	231.3	1359
7	9.61×10^{-7}	8.55×10^{-8}	1.14	2.41
8	9.39×10^{-5}	8.38×10^{-6}	71.1	234.8

3) in all experiments value f_1 remained fixed ($f_1 = 1.991$);

4) jitter RMSD σ_τ was specified in fractions of the input signal period at frequency $F_S f_1$ (F_S is the sampling frequency);

5) RMSD of additive noise σ_η was specified in ADC quanta and was the same in all experiments (σ_η = 1.1);

6) sample distributions of additive noise and jitter were simulated in terms of the normal distribution. 100 samplings were used to obtain one pair of estimates $\hat{\sigma}_\tau^2$ and $\hat{\sigma}_\eta^2$. The length of each sampling was equal to 100 samples.

Table 3 presents the following results with regard to the multifrequency method: sampling average $E\{\hat{\sigma}_\tau^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\tau^2\}}$ for $\hat{\sigma}_\tau^2$, sampling average $E\{\hat{\sigma}_\eta^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\eta^2\}}$ for $\hat{\sigma}_\eta^2$. The sampling average and RMSD were estimated on the basis of a set of 1000 experiments.

From the results of Table 3 we can draw a conclusion that the estimation accuracy depends on quantities $|\sin(2\pi f_1 N)\sin^{-1}(2\pi f_1)|, |\sin(2\pi f_2 N)\sin^{-1}(2\pi f_2)|$ and is determined by a larger one of them. In this case we did not detect the dependence of the accuracy on values of quantities

$$|\sin(2\pi(f_1 \pm f_2)N)\sin^{-1}(2\pi(f_1 \pm f_2))|.$$

Table 4 presents the results of simulation for the case of a 4-channel system. The conditions of experiment in each channel of the system corresponded to the parameters specified in Table 2. Table 4 presents the sampling average $E\{\hat{\sigma}_\tau^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\tau^2\}}$ for $\hat{\sigma}_\tau^2$ in each channel during their estimation by using the single-channel multifrequency method and also the sampling average $E\{\hat{\sigma}_\tau^2\}$ and RMSD $\sqrt{D\{\hat{\sigma}_\tau^2\}}$ during the estimation of $\hat{\sigma}_\tau^2$ by the multichannel method. The sampling average and RMSD were still estimated on the basis of a set of 1000 experiments.

From the results of Table 4 we can draw a conclusion that in the case of multichannel estimates (under the assumptions made with respect to noises) estimate dispersion $\hat{\sigma}_\tau^2$ is inversely proportional to the square root of the number of channels.

For comparing the results of simulation with real systems we shall consider a 16-bit ADC of model AD9467 (Analog Devices Company) having the maximum sampling frequency of 250 MHz and the maximum bandwidth of analog input signal of 900 MHz. In this case the absolute value of jitter RMSD equal

Table 4

No.	Channel 1		Channel 2		Channel 3		Channel 4		4-Channel	
	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$	$E\{\hat{\sigma}_\tau^2\}$	$\sqrt{D\{\hat{\sigma}_\tau^2\}}$
1	9.8×10^{-7}	1.3×10^{-7}	9.9×10^{-7}	1.3×10^{-7}	9.8×10^{-7}	1.3×10^{-7}	9.8×10^{-7}	1.3×10^{-7}	9.8×10^{-7}	6.6×10^{-8}
2	9.7×10^{-5}	1.3×10^{-5}	9.8×10^{-5}	1.3×10^{-5}	9.8×10^{-5}	1.3×10^{-5}	9.7×10^{-5}	1.3×10^{-5}	9.8×10^{-5}	6.4×10^{-6}
3	9.7×10^{-7}	5.5×10^{-8}	9.7×10^{-7}	5.8×10^{-8}	9.7×10^{-7}	6.0×10^{-8}	9.7×10^{-7}	5.9×10^{-8}	9.7×10^{-7}	3.0×10^{-8}
4	8.9×10^{-5}	6.4×10^{-6}	8.9×10^{-5}	6.1×10^{-6}	8.9×10^{-5}	6.1×10^{-6}	8.9×10^{-5}	6.6×10^{-6}	8.9×10^{-5}	3.3×10^{-6}
5	9.5×10^{-7}	4.5×10^{-7}	9.6×10^{-7}	4.6×10^{-7}	9.8×10^{-7}	4.7×10^{-7}	9.6×10^{-7}	4.6×10^{-7}	9.6×10^{-7}	2.4×10^{-7}
6	8.8×10^{-5}	4.3×10^{-5}	8.6×10^{-5}	4.3×10^{-5}	8.9×10^{-5}	4.3×10^{-5}	8.7×10^{-5}	4.3×10^{-5}	8.7×10^{-5}	2.2×10^{-5}
7	9.6×10^{-7}	8.7×10^{-8}	9.7×10^{-7}	8.5×10^{-8}	9.6×10^{-7}	8.7×10^{-8}	9.6×10^{-7}	8.8×10^{-8}	9.6×10^{-7}	4.4×10^{-8}
8	9.4×10^{-5}	8.8×10^{-6}	9.4×10^{-5}	8.6×10^{-6}	9.4×10^{-5}	8.5×10^{-6}	9.4×10^{-5}	8.5×10^{-6}	9.4×10^{-5}	4.3×10^{-6}

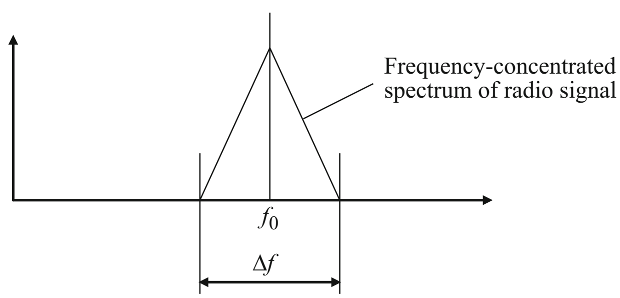

Fig. 1.

to 1 ns (the value attainable by using cheap clock pulse generators) at the frequency of input harmonic signal equal to 874 MHz corresponds to 8.74×10^{-4} of the input signal period and the ratio of the signal frequency to the sampling frequency 874 MHz/250 MHz = 3.496. Taking into account that the ADC intrinsic noise approximately amounts to 0.88 quantum, we assume that the aggregate additive noise in the process of measurement is equal to 1.5 quantum of ADC. The specified conditions correspond to the conditions of single-frequency experiment No. 10 from Table 1. Thus, the results of numerical simulation corroborate the operability of the developed method in the conditions adequate to the existing technical systems.

The simulation was performed by using the input signal frequencies higher than the sampling frequency due to the following reasons:

1. The development and analysis of radio engineering systems operating in the subdiscretization mode (i.e. when the input signal frequency is higher than a half of the sampling frequency) is one of the areas of interest for the authors of this paper.

2. The use of the subdiscretization mode is common in technologies of the wireless telecommunications and makes it possible to simplify the radio channel (reducing the number of conversions and easing the requirements to filters) without increasing the data flow that requires digital processing.

3. The working frequency band of state-of-the-art fast-acting ADC, as a rule, is several times as high as the maximum sampling frequency, i.e., these ADC are oriented on operation in the subdisretization mode.

4. The higher is the input signal frequency, the stronger is the manifestation of jitter effects.

5. The digitization of radio signals with sampling frequency that is less than the central frequency of their spectrum ($F_d \geq 2\Delta f$) corresponds to the well-known interpretation of the Kotelnikov theorem for

radio-frequency signals under the condition that the information about the signal is contained not in its central frequency, but in the signal spectrum displaced by this frequency and localized in terms of the width (Fig. 1).

In conclusion it should be noted that according to the results of numerical simulation the developed methods make it possible to perform the estimation of jitter dispersion without a need of synchronizing the input signal generator and the clock pulse generator of ADC and thus ensure the performance of measurements under conditions when synchronization of generators is not possible. The multichannel method can be used in digital antenna arrays, for example, as one of the methods of built-in diagnostics. The comparison of these methods with the methods for coherent systems and the investigation of the accuracy of the resultant estimates are the subject of further studies.

REFERENCES

1. M. V. Bondarenko, "Phase Method of Estimation of Aperture Uncertainty Time," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **53**(1), 48 (2010) [Radioelectron. Commun. Syst. **53**(1), 42 (2010)].
2. F. Verbeyst, Y. Rolain, R. Pintelon, and J. Schoukens, "Enhanced Time Base Jitter Compensation of Sine Waves," in *Proc. of Instrumentation and Measurement Technology Conference* (*IMTC 2007*), *May 1–3*, *2007*, *Warsaw*, *Poland* (Warsaw, 2007), pp. 1–5.
3. I. S. Gradshtein and I. M. Ryzhik, *Tables of Integrals*, *Sums*, *Series*, *and Products* (Fizmatgiz, Moscow, 1963) [in Russian].
4. V. I. Slyusar, RF Patent No. 2054684, Byull. Izobret., No. 5 (1996).
5. V. I. Slyusar and V. G. Smolyar, "The Method of Nonorthogonal Frequency-Discrete Modulation of Signals for Narrow-Band Communication Channels," Izv. Vyssh. Uchebn. Zaved., Radioelektron. **47**(4), 53 (2004); Radioelectron. Commun. Syst. **47**(4), 47 (2004).
6. V. A. Varyukhin, *Fundamental Theory of Multichannel Analysis* (VA PVO SV, Kyiv, 1993) [in Russian].